09

Light is a non-mechanical form of energy (i.e. it requires no medium for propagation), due to which we have sensation of vision. Light always travels in a straight line and its speed is very high. In vacuum, light travels with a speed of 3×10^8 m/s.

RAY OPTICS AND OPTICAL INSTRUMENTS

|TOPIC 1|
Ray Optics

A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a ray of light and a bundle of such rays constitutes a beam of light. The branch of study of light is called **Optics**.

Broadly optics is divided into three groups

(i) Geometrical optics (Ray optics)
(ii) Wave optics
(iii) Quantum optics

CHAPTER CHECKLIST

- Ray Optics
- Refraction
- Refraction at Spherical Surfaces and Lenses
- Prism and Optical Instruments

GEOMETRICAL OPTICS (RAY OPTICS)

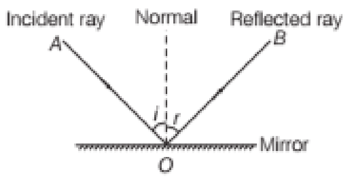
In this, light is considered as a ray which travels in straight line. Geometrical optics states that for each and every object, there is an image. It works on following assumptions:

(i) Rectilinear propagation of light, i.e. light ray travels in straight line.
(ii) Laws of reflection.
(iii) Laws of refraction.
(iv) Physical independence of light rays, i.e. two light rays are totally independent of each other.

Reflection of Light

Reflection is the phenomenon of change in the path of light without any change in the medium.

The returning back of light in the same medium from which it has come after striking a surface is called **reflection of light**.



The incident ray reflected ray and the normal to the reflecting surface lie in the same plane

Laws of Reflection

The laws of reflection are as given below

(i) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.

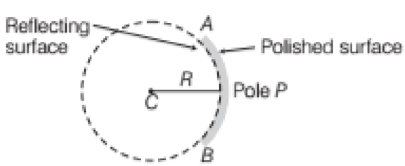
(ii) The angle of reflection (r) is equal to the angle of incidence (i), i.e. $i = r$. For normal incidence, $\angle i = 0°$, therefore $\angle r = 0°$. Hence, a ray of light falling normally on a mirror, retraces its path on reflection.

Spherical Mirrors

Spherical mirror is a mirror whose reflecting surface is a part of a hollow sphere. Spherical mirrors are of two types

Concave Spherical Mirror

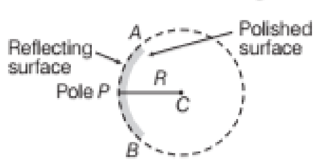
A spherical mirror whose reflecting surface is towards the centre of the sphere is called **concave spherical mirror**.



Concave mirror

Convex Spherical Mirror

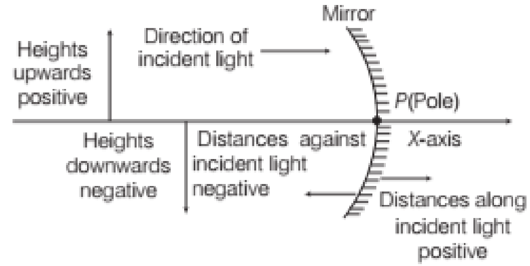
A spherical mirror whose reflecting surface is away from the centre of the sphere is called **convex spherical mirror.**



Convex mirror

Sign Convention

To derive the relevant formulae for reflection by spherical mirrors and refraction by spherical lenses (which we will study later in this chapter), we must first adopt a sign convention for measuring distances.



The cartesian sign convention

According to the cartesian sign convention,

(i) All the distances are measured from pole (P) of the mirror or the optical centre(O) of the lens.

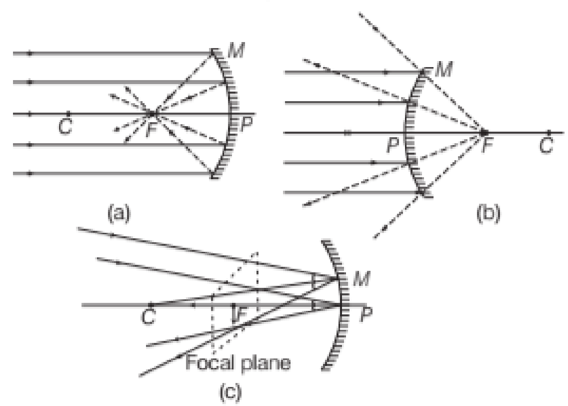
(ii) The principal axis of the mirror or lens is taken as X-axis and the pole or optical centre as origin.

(iii) Distances measured in the direction of the incident light are taken as positive and opposite to the direction of incident light as negative.

(iv) The heights measured upwards with respect to X-axis and normal to the principal axis of the mirror or lens are taken as positive and the heights measured downwards are taken as negative.

Focal Length of Spherical Mirrors

When a parallel beam of light is incident on a concave or convex mirror, the reflected rays converge or appear to diverge from a point F on principal axis called **principal focus** of the mirror. We assume that the rays are paraxial, i.e. they are incident at points close to the pole P of the mirror and make small angles with the principal axis.



Focus of a concave and convex mirrors

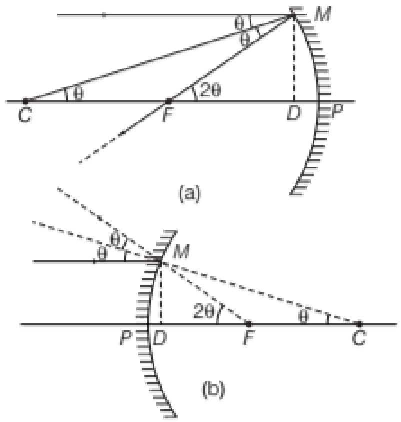
If the paraxial beam of light were incident, making some angle with principal axis, the reflected rays would converge or appear to diverge from a point in a plane through F normal to the principal axis. This is called the **focal plane** of the mirror.

The relation between Focal Length (f) and radius of Curvature (R)

$$\boxed{f = \frac{R}{2}}$$

Proof

Consider a ray parallel to the principal axis striking the mirror at point M. Then CM will be perpendicular to the mirror at point M. Let θ be the angle of incidence and MD be perpendicular to the principal axis.



Geometry of reflection of an incident ray on
(a) concave spherical mirror and
(b) convex spherical mirror

Then, $\angle MCP = \theta$

and $\angle MFP = 2\theta$

Now, $\tan\theta = \dfrac{MD}{CD}$

and $\tan 2\theta = \dfrac{MD}{FD}$ ...(i)

For small θ (condition true for paraxial rays), $\tan\theta \approx \theta$ and $\tan 2\theta \approx 2\theta$

Therefore, from Eq.(i), we get

$$\frac{MD}{FD} = 2\frac{MD}{CD} \quad \text{or} \quad FD = \frac{CD}{2} \qquad \text{...(ii)}$$

Again, for small θ, we can observe that the point D is very close to the point P. Therefore, $FD = f$ and $CD = R$.

From Eq.(ii), we have

$$\boxed{f = \frac{R}{2}}$$

Real Image If rays emanating from a point actually converge at another point after reflection/refraction, that point is called the **real image** of the first point.

Virtual Image If rays emanating from a point do not actually meet but appear to diverge from another point, that point is called the **virtual image** of the first point.

Mirror Formula

Mirror formula (or equation) is a relation between focal length of the mirror and distances of object and image from the mirror.

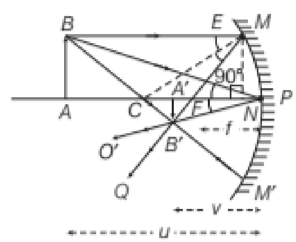
In principle, we can take any two rays originating from a point on an object, trace their paths, find their point of intersection and thus, obtain the image of the point due to reflection at a spherical mirror. However in practice, it is convenient to choose any two of the following rays.

(i) The ray from the point, which is parallel to the principal axis after reflection goes through the focus of the mirror.

(ii) The ray passing through the centre of curvature of a concave mirror or appearing to pass through it for a convex mirror simply retraces the path.

(iii) The ray passing through (or directed towards) the focus of the concave mirror or appearing to pass through (or directed towards) the focus of a convex mirror after reflection is parallel to the principal axis.

(iv) The ray incident at any angle at the pole is reflected following the laws of reflection.



In the above figure, the ray diagram is considering three rays for image formation by a concave mirror. In the figure, triangles $A'B'F$ and NEF are similar.

Then, $\dfrac{A'B'}{NE} = \dfrac{A'F}{NF}$

As, the aperture of the concave mirror is small and the points N and P lie very close to each other, then

$NF \approx PF$

and $NE = AB$.

$\Rightarrow \dfrac{A'B'}{AB} = \dfrac{A'F}{PF}$

Since, all the distances are measured from the pole of the concave mirror, we have

$$A'F = PA' - PF$$

$$\therefore \qquad \frac{A'B'}{AB} = \frac{PA' - PF}{PF} \qquad \ldots\text{(i)}$$

Also, triangles ABP and $A'B'P$ are similar, then

$$\frac{A'B'}{AB} = \frac{PA'}{PA} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{PA' - PF}{PF} = \frac{PA'}{PA} \qquad \ldots\text{(iii)}$$

Applying new Cartesian sign convention, we have

$$PA = -u$$

[$\because$ distance of object is measured against incident ray]

$$PA' = -v$$

[$\because$ distance of image is measured against incident ray]

$$PF = -f$$

[$\because$ focal length of concave mirror is measured against incident ray]

Substituting these values in Eq. (iii), we have

$$\frac{-v-(-f)}{-f} = \frac{-v}{-u} \Rightarrow \frac{v-f}{f} = \frac{v}{u} \Rightarrow \frac{v}{f} - 1 = \frac{v}{u}$$

Dividing both sides by v, we get

$$\therefore \qquad \boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

The above relation is called **mirror formula.**

Relation between u, v and R

$\because$ Focal length of the mirror, $f = \dfrac{R}{2}$

$$\therefore \qquad \frac{1}{u} + \frac{1}{v} = \frac{1}{R/2} \Rightarrow \boxed{\frac{1}{u} + \frac{1}{v} = \frac{2}{R}}$$

Important Points

(i) An object is always placed in front of a spherical mirror, so the distance of the object (u) is always negative.

(ii) In a spherical mirror, real image is formed in front of the mirror. Therefore, its distance (v) is taken as negative. However, virtual image is formed at the back of the mirror. So, its distance (v) is taken as positive, as per the new Cartesian sign convention.

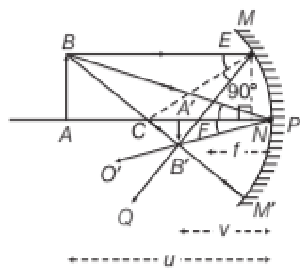
(iii) Similarly, focal length of a concave mirror is taken as negative, while that of convex mirror is taken as positive.

LINEAR MAGNIFICATION

The ratio of the height of the image (h') formed by a spherical mirror to the height of the object (h) is called the linear magnification produced by the spherical mirror.

It is denoted by m, i.e.

$$m = \frac{h'}{h}$$



In the above figure, triangles ABP and $A'B'P$ are similar.

$$\frac{A'B'}{AB} = \frac{PA'}{PA}$$

Applying new Cartesian sign conventions, we have

$A'B' = -h'$ [$\because$ height of image measured downward]

$AB = +h$ [$\because$ height of object measured upward]

$PA = -u$ [$\because$ object distance measured against incident ray]

$PA' = -v$ [$\because$ image distance measured against incident ray]

The above equation becomes

$$\frac{-h'}{h} = \frac{-v}{-u}$$

or

$$\frac{h'}{h} = \frac{-v}{u} \qquad \ldots\text{(iv)}$$

$$\therefore \qquad \boxed{\text{Linear magnification, } m = \frac{h'}{h} = -\frac{v}{u}}$$

The expression for magnification is same for both the concave and convex mirrors.

(i) When $m > 1$, image formed is enlarged.

(ii) When $m < 1$, image formed is diminished.

(iii) When m is positive, image must be erect, i.e. virtual.

(iv) When m is negative, image must be inverted, i.e. real.

(v) In case of concave mirror, m can be either positive or negative but in case of convex mirror, m is positive only.

EXAMPLE |1| A candle flame is held 3 cm away from a concave mirror of radius of curvature 24 cm. Where is the image formed? What is the nature of the image?

Sol. Given, object distance, $u = -3$ cm

Radius of curvature, $R = -24$ cm

$$\therefore \quad f = \frac{R}{2} = \frac{-24}{2} = -12 \text{ cm}$$

According to mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{(-12)} - \frac{1}{-3}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{-12} + \frac{1}{3} = \frac{-1+4}{12}$$

$$\Rightarrow \quad v = 4 \text{ cm}$$

$$\because \quad \text{Magnification, } m = -\frac{v}{u} = \frac{-4}{-3} = +1.33$$

i.e. The image formed is virtual, erect and magnified.

EXAMPLE |2| An object is placed in front of a convex mirror of focal length 30 cm. If the image is a quarter of the size of the object, find the position of the image.

Sol. Given, focal length, $f = +30$ cm

Magnification, $m = \frac{1}{4}, v = ?$

From mirror's formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad \left[\because m = -\frac{v}{u} \Rightarrow u = -\frac{v}{m}\right]$$

$$\Rightarrow \quad \frac{1}{f} = -\frac{m}{v} + \frac{1}{v}$$

$$\Rightarrow \quad m = \frac{f-v}{f} \Rightarrow \frac{1}{4} = \frac{30-v}{30}$$

$$\Rightarrow \quad 30 = 120 - 4v$$

$$\Rightarrow \quad v = \frac{90}{4} = +22.5 \text{ cm}$$

As, v is positive, therefore a virtual and erect image will be formed on other side of the object.

Uses of Spherical Mirrors

(i) Convex mirror is used as a reflector in street lamps to diverge the light over a large area.

(ii) Convex mirror is used as rear view mirror (or driver's mirror) in vehicles, because it has a wider field of view.

(iii) Concave mirror is used as a reflector in search light, head lights of vehicles, etc.

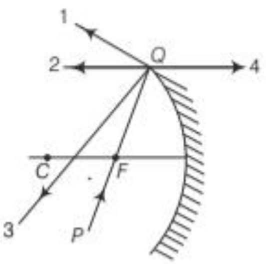
(iv) Concave mirror is also used as face looking mirror because it forms erect and magnified image.

(v) Spherical mirrors are also used as trick mirrors.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (figure). Which of the four rays correctly shows the direction of reflected ray? **NCERT Exemplar**



(a) 1 (b) 2 (c) 3 (d) 4

2. In reflection over a spherical mirror, ray parallel to principal axis, after reflection from mirror pass through

(a) focus (b) centre of curvature
(c) pole of mirror (d) any point

3. A ray passing through or directed towards centre of curvature of a spherical mirror is reflected such that it trace back of its path, because

(a) it does not follow law of reflection
(b) angle of incidence is 0°
(c) centre of curvature is midway between object and pole
(d) distance of centre of curvature from focus is equal to its distance from pole

4. If lower half of a concave mirror is blackened, then

(a) image distance increases
(b) image distance decreases
(c) image intensity increases
(d) image intensity decreases

5. An object 2 cm high is placed at a distance of 16 cm from a concave mirror, which produces a real image 3 cm high. What is the focal length of the mirror?

(a) – 9.6 cm (b) – 3.6 cm
(c) – 6.3 cm (d) – 8.3 cm

VERY SHORT ANSWER Type Questions

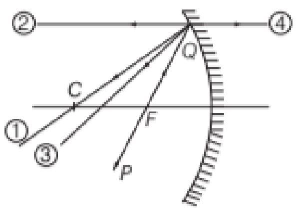
6. A mirror is turned through 15°. With what angle will the reflected ray turn?

7. A thick plane mirror forms a number of images of a point source of light. Which image is the brightest?

8. A boy is running towards a plane mirror with a speed of 2 m/s. With what speed the image of the boy approach him? **Foreign 2011**

9. How can the real image of an object be obtained with a convex mirror? **Delhi 2011**

10. The direction of ray of light incident on a concave mirror is shown by *PQ* while directions in which the ray would travel after reflection is shown by four rays marked as 1, 2, 3 and 4 in the figure? Which of the four rays correctly shows the direction of reflected rays?



11. Give the effect on image, if lower half of the concave mirror is blackened.

SHORT ANSWER Type Questions

12. A concave mirror of small aperture forms a sharper image. Why?

13. Choose the statement as wrong or right and justify.

(i) Linear magnification of a spherical mirror is given by $\frac{v}{u}$.

(ii) Focal length of plane mirror is zero.

(iii) $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ can be applied to all types of mirror.

14. Use the mirror equation to show that an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$. **All India 2015**

15. You read a newspaper because of the light that it reflects. Then, why do not you see even a faint image of yourself in the newspaper?

16. "Mirrors used in search lights are parabolic but not concave spherical". Explain, why?

17. A short object of length L is placed along the principal axis of a concave mirror away from focus. The object distance is u. If the mirror has a focal length f, what will be the length of the image? You may take $L << |v-f|$.

NCERT Exemplar

> **Hints:** The length of image is the separation between the images formed by mirror of the extremities of object.

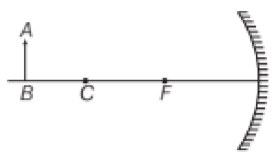
LONG ANSWER Type I Questions

18. (i) A mobile phone lies along the principal axis of a concave mirror. Show with the help of a suitable diagram the formation of its image. Explain, why magnification is not uniform?

(ii) Suppose the lower half of the concave mirror's reflecting surface is covered with an opaque material. What effect this will have on the image of the object? Explain.

Delhi 2014

19. An object *AB* is kept in front of a concave mirror as shown in the figure.



(i) Complete the ray diagram showing the image formation of the object.

(ii) How will the position and intensity of the image be affected, if the lower half of the mirror's reflecting surface is painted black?

All India 2012

20. An infinitely long rod lies along the axis of concave mirror of focal length f. The near end of the rod is at a distance $x > f$ from the mirror f. Then, what will be the length of the image of the rod?

21. Show that spherical mirror formula is applicable to a plane mirror.

22. Use the mirror equation to show that

(i) an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

(ii) a convex mirror always produces a virtual image independent of the location of the object.

(iii) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image. **All India 2011**

23. (a) Draw a ray diagram to show image formation when the concave mirror produces a real, inverted and magnified image of the object.

(b) Obtain the mirror formula and write the expression for the linear magnification. **CBSE 2018**

LONG ANSWER Type II Questions

24. State and derive mirror formula for a concave mirror. State the sign convention used.

25. Use the mirror equation to deduce that,

(i) an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

(ii) a convex mirror always produces a virtual image independent of the location of the object.

(iii) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(iv) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image. **NCERT**

NUMERICAL PROBLEMS

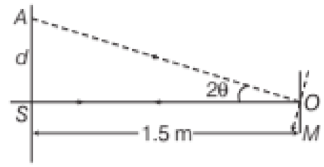
26. A square wire of side 3 cm is placed 25 cm away from a concave mirror of focal length 10 cm. What is the area enclosed by the image of the wire? Given, the centre of the wire is on the axis of the mirror, with its two sides normal to the axis.

27. An erect image 3 times the size of the object is obtained with a concave mirror of radius of curvature 36 cm. What is the position of the object?

28. A 12 m tall tree is to be photographed with a pin hole camera. It is situated 15 m away from the pin hole. How far should the screen be placed from the pin hole to obtain a 12 cm tall image of the tree?

29. Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

30. Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in the figure. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away? **NCERT**



31. Suppose while sitting in a parked car, you notice a Jogger approaching towards you in the rear view mirror having $R = 2$ m. If the Jogger is running at a speed of 5 m/s, how fast is the image of Jogger moving, when the Jogger is

(i) 39 m

(ii) 29 m

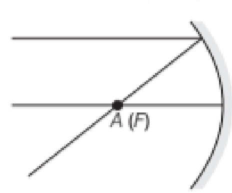
(iii) 19 m

(iv) 9 m away ?

HINTS AND SOLUTIONS

1. (b) The PQ ray of light passes through focus F and incident on the concave mirror, after reflection, should become parallel to the principal axis and shown by ray-2 in the figure.

2. (a) Parallel beam passes through focus after reflection. This can be shown in the figure given below.



3. (b) As we know, angle $i = 0°$ and angle $r = 0°$, when light ray is passes through centre of curvature of a spherical mirror is reflected such that it trace back its path.

4. (d) If lower half of a concave mirror is blackened, then image will be now only half of the object, but taking the laws of reflection to be true for all points of the remaining part of the mirror, the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will be low i.e., half.

5. (a) Here, $h_1 = 2$ cm, $u = -16$ cm $h_2 = -3$ cm

(since image is real and inverted)

$\therefore \quad m = \frac{-h_2}{h_1} = \frac{v}{u}$

$\Rightarrow \quad v = \frac{-h_2}{h_1}u = \frac{3}{2} \times (-16) = -24$ cm

Now, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = -\frac{1}{24} - \frac{1}{16}$

$\Rightarrow \frac{-2-3}{48} = \frac{-5}{48} \Rightarrow f = \frac{-48}{5} = -9.6$ cm

6. The reflected ray turns twice the angle through which mirror is turned, i.e. 30°.

7. A thick plane mirror consist of two surfaces (top and bottom), where the reflection takes place. The images are formed after reflection from both the surfaces, except for the first image. The second image is the brightest of all as minimum absorption takes place and bounces of the silvery layers which makes the bottom surface.

8. The image of the object in a plane mirror is as far behind the mirror as the object is in front of it. Therefore, the image of the boy comes near the mirror through the distance equal to that moved by the boy towards the plane mirror. Hence, the image of the boy will approach him with double his speed, i.e. with 4 m/s.

9. A convex mirror produces a real image of a virtual object. Therefore, if a beam of light from a virtual object converges to a point behind the convex mirror, then its real image will be formed in front of the mirror.

10. The incident ray PQ passes through the focus, so the reflected ray is parallel to the principal axis. So, the answer is ray 2.

11. If the lower half of the concave mirror is blackened, then there is no change in the position of image, only intensity will get reduced.

12. The rays of light travelling parallel to the principal axis after reflection from a concave mirror meet at a single point only, if the beam of light is narrow or if the mirror is of small aperture. In case, a wide beam of light falls on a concave mirror of large aperture, the rays after reflection from the mirror do not come to focus at a single point. Therefore, it follows that, if the aperture of the concave mirror is small, the image formed will be sharper.

13. (i) Wrong, linear magnification of spherical mirror is $-\frac{v}{u}$ (using sign conventions).

(ii) Wrong, as the plane mirror can be considered to be the limit of either a concave or convex spherical curved mirror as the radius, therefore the focal length of plane mirror becomes infinite.

(iii) Right, but for plane mirror using this formula, focal length becomes infinite.

14. According to the mirror equation, we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

where, u = distance of the object from the mirror,

v = distance of the image from the mirror

and f = focal length of the mirror.

From the mirror equation, we have

$$v = \frac{uf}{u-f} \qquad \text{...(i)}$$

Applying new cartesian sign convention, we get

$f = -ve$ and $u = -ve$

Given, $f < u < 2f$

$\Rightarrow \quad v = -ve$ [from Eq. (i)]

Magnification is given by $m = -\left(\frac{-v}{-u}\right) = -ve$

Hence, the image formed is real.

From the mirror formula, when $u = -2f$.

$$\Rightarrow \quad \frac{1}{-2f} + \frac{1}{v} = \frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{2f} - \frac{1}{f} = \frac{-1}{2f}$$

When the object is at f, then image is formed at infinity. This shows that when $f < u < 2f$, then $\infty > v > 2f$.

15. We can an image, if it is caused by regular reflection. In the case of newspaper, the inhomogeneities of the surface cause diffuse reflection. So, the incident parallel beam is scattered in all directions, hence no image is seen.

16. A search light produces an intense parallel beam of light. This require a reflector of large aperture. When a source is placed at the focus of large concave mirror only the paraxial rays are reflected as parallel beam but when a source is placed at the focus of parabolic mirror.

All the rays are reflected as an intense parallel beam.

17. Since, the object distance is u. Let us consider the two ends of the object be at distance $u_1 = u - L/2$ and $u_2 = u + L/2$ respectively, so that $|u_1 - u_2| = L$. Let the image of the two ends be formed at v_1 and v_2 respectively, so that the image length would be

$$L' = |v_1 - v_2| \qquad \text{...(i)}$$

Applying mirror formula, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \text{or } v = \frac{fu}{u-f}$$

On solving, the positions of two images are given by

$$v_1 = \frac{f(u - L/2)}{u - f - L/2}, \; v_2 = \frac{f(u + L/2)}{u - f + L/2}$$

For length, substituting these values in Eq. (i), we get

$$L' = |v_1 - v_2| = \frac{f^2 L}{(u-f)^2 - L^2/4}$$

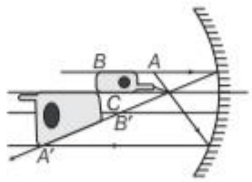
Since, the object is short and kept away from focus, we have

$$L^2/4 << (u-f)^2$$

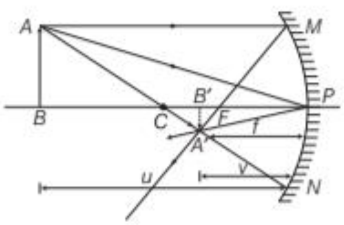
Hence, finally, $L' = \frac{f^2}{(u-f)^2} L$

This is the required expression of length of an image.

18. (i) The ray diagram for the formation of the image of the mobile phone is shown below. The image of the part which is on the plane perpendicular to principal axis will be on the same plane. It will be of the same size, i.e. $B'C = BC$

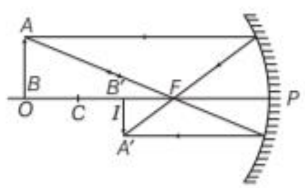


(ii) We may think that the image will now show only half of the object, but considering the laws of reflection to be true for all points of the remaining part of the mirror, the image will be that of the whole object.



However, as the area of the reflecting surface has been reduced, the intensity of the image will be low, i.e. half.

19. (i) The ray diagram showing the image formation of the object.



(ii) The position of image is unaffected but the intensity of image is reduced.

20. Using mirror formula, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Here, $u = -u, v = ?, f = -f$

$\therefore$ $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

or $\frac{1}{v} = \frac{1}{-f} + \frac{1}{u}$

$\Rightarrow$ $v = \frac{-fu}{-f+u}$

$\therefore$ Length of the image $= \frac{-fu}{-f+u} - f$

$$= \frac{-fu + f^2 - fu}{-f+u} = \frac{f^2}{u-f}$$

21. The spherical mirror formula is given by

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \text{...(i)}$$

For a plane mirror, $R = \infty$

$\therefore$ $f = \frac{R}{2} = \infty$

From Eq. (i), we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{\infty} = 0 \Rightarrow \frac{1}{v} = \frac{-1}{u} \Rightarrow v = -u$$

As, u is negative, v becomes positive.

Thus, image is formed behind the mirror at the same distance as the object is in front of it. This happens in a plane mirror and is the desired result. Also, note that magnification, $m = -\frac{v}{u}$ is 1.

22. (i) Refer to Q. 14 on page 354.

(ii) For convex mirror, $f > 0$

Also, $u < 0$

But from mirror equation,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} + \frac{1}{|-u|} \quad \text{[taking } u \text{ with sign]}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

If f and u to be positive, then $\frac{1}{v} > 0 \Rightarrow v > 0$

Hence, virtual image is formed.

(iii) For concave mirror,

$$f < 0, u < 0, |f| > |u| > 0$$

But from mirror equation,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{-1}{|f|} = \frac{1}{v} - \frac{1}{|u|}$$

$$\frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$$

$\because$ $|v| < |f| \Rightarrow \frac{1}{|u|} > \frac{1}{|f|}$

$\Rightarrow$ $\frac{1}{v} > 0 \Rightarrow v > 0$

Image is formed on RHS of mirror, i.e. virtual image.

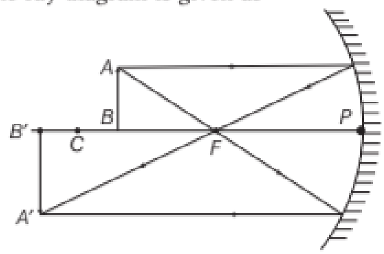
Also, $\frac{1}{f} = \frac{1}{|v|} - \frac{1}{|u|}$

For concave mirror, f is negative.

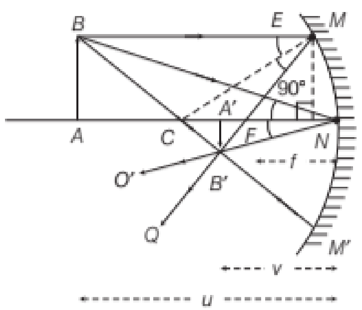
$\Rightarrow$ $\frac{1}{|v|} < \frac{1}{|u|} \Rightarrow \frac{|v|}{|u|} > 1 \Rightarrow m > 1$

Enlarged virtual image formed on the other side of mirror.

23. (a) Concave mirror form real, inverted and magnified image of an object when it is placed between C and F. The ray diagram is given as



(b) In the given figure, the ray diagram considering three rays for image formation by a concave mirror.



In the figure, triangles $A'B'F$ and NEF are similar.

Then, $$\frac{A'B'}{NE} = \frac{A'F}{NF}$$

As the aperture of the concave mirror is small, the points N and P lie very close to each other.

$$NF = PF \text{ and } NE = AB$$

$$\frac{A'B'}{AB} = \frac{A'F}{PF}$$

Since, all the distances are measured from the pole of the concave mirror, we have

$$A'F = PA' - PF$$

$$\therefore \quad \frac{A'B'}{AB} = \frac{PA' - PF}{PF} \quad \text{...(i)}$$

Also, triangles ABP and $A'B'P$ are similar, then

$$\frac{A'B'}{AB} = \frac{PA'}{PA} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{PA' - PF}{PF} = \frac{PA'}{PA} \quad \text{...(iii)}$$

Applying new Cartesian sign convention, we have

$PA = -u$ ($\because$ distance of object is measured against incident ray)

$PA' = -v$

($\because$ distance of image is measured against incident ray)

$PF = -f$

($\because$ focal length of concave mirror is measured against incident ray)

Substituting these values in Eq. (iii), we get

$$\frac{-v-(-f)}{-f} = \frac{-v}{-u}$$

$$\Rightarrow \quad \frac{v-f}{f} = \frac{v}{u} \Rightarrow \frac{v}{f} - 1 = \frac{v}{u}$$

Dividing both sides by v, we get

$$\therefore \quad \boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

The above relation is called **mirror formula.**

Linear magnification

The ratio of the height of the image (h') formed by a spherical mirror to the height of the object (h) is called the linear magnification produced by the spherical mirror.

It is denoted by m.

$$m = \frac{h'}{h}$$

24. Refer to the text on pages 351 and 352.

25. For parts (i), (ii) and (iv), refer to Q. 22 on page 354.

(iii) For convex mirror, $f > 0, u < 0$

From mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

So, $$\frac{1}{v} > \frac{1}{f} \text{ or } v < f$$

Also, $$\frac{1}{v} > \frac{-1}{u} \text{ or } \frac{-v}{u} < 1,$$

i.e. $m < 1$

Thus, image is always located between pole and focus of the mirror and is always diminished in size.

26. Here, $u = -25$ cm, $f = -10$ cm

Using mirror formula, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$-\frac{1}{10} = -\frac{1}{25} + \frac{1}{v}$$

$$\Rightarrow \quad v = \frac{-50}{3}$$

Now, magnification, $m = \frac{-v}{u} = -\left[\frac{\left(-\frac{50}{3}\right)}{(-25)}\right] = -\frac{2}{3}$

Length and breadth both change in the same proportion.

Area of the object, $A_o = 3 \times 3 = 9 \text{ cm}^2$

$$\therefore \quad \frac{A_i}{A_o} = \left(-\frac{2}{3}\right)^2$$

$$\Rightarrow \quad A_i = \left(\frac{4}{9}\right) \times 9 = 4 \text{ cm}^2$$

27. Given, magnification, $m = +3$, $R = -36$ cm

Object distance, $u = ?$

Let $u = -x$

$$m = \frac{h_2}{h_1} = \frac{+v}{-u} = 3$$

$\Rightarrow$ $v = -3u \Rightarrow v = 3x$

Applying mirror formula, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R} \Rightarrow \frac{1}{-x} + \frac{1}{3x} = \frac{2}{-36}$$

$$\Rightarrow \frac{-3+1}{3x} = \frac{-1}{18} \Rightarrow 3x = 36$$

$\Rightarrow$ $x = 12$ cm or $u = -12$ cm

28. Given, $h_1 = 12$ m, $u = -15$ m, $v = ?$,

$h_2 = 12$ cm $= 0.12$ m (symbols have their usual meanings)

As, $\frac{h_2}{h_1} = -\frac{v}{u}$

$$\Rightarrow v = -\frac{h_2}{h_1} \times u$$

$$= -\frac{0.12}{12} \times -15 = 0.15 \text{ m} = 15 \text{ cm}$$

Thus, the screen should be placed 15 cm from the pin hole to obtain a 12 cm tall image of the tree.

29. Given, $\lambda = 5000$ Å $= 5 \times 10^{-7}$ m

Frequency of incident light,

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{5 \times 10^{-7}} \quad [\because c = 3 \times 10^8 \text{ m/s}]$$

$$= 6 \times 10^{14} \text{ Hz}$$

On reflection, there is no change in wavelength or frequency. Therefore, $\lambda' = \lambda = 5000$ Å

or $\nu' = \nu = 6 \times 10^{14}$ Hz

For reflected ray to be normal to incident ray,

$$i + r = 90^\circ \Rightarrow i + i = 90^\circ$$

$$\Rightarrow i = \frac{90^\circ}{2} = 45^\circ$$

30. Given, deflection of the mirror, $\theta = 3.5^\circ$

Distance between screen and mirror, $x = 1.5$ m

As we know that when mirror turns by angle θ, the reflected light may turn by 2θ.

$$2\theta = 2 \times 3.5^\circ = 7^\circ = \frac{7\pi}{180} \text{ radians}$$

Again, in ΔAOS, $\tan 2\theta = \frac{AS}{SM}$

$$\tan\left(\frac{7\pi}{180}\right) = \frac{AS}{1.5} = \frac{d}{1.5}$$

or $$d = 1.5 \tan\left(\frac{7\pi}{180}\right)$$

For small angle, $\tan \frac{7\pi}{180} = \frac{7\pi}{180}$

$$d = 1.5 \times \frac{7\pi}{180} = 0.18 \text{ m}$$

31. Here, $R = 2$ m, $f = \frac{R}{2} = \frac{2}{2} = 1$ m

Using mirror formula, we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{v} = \frac{u-f}{fu}$$

$$\Rightarrow v = \frac{fu}{u-f} \quad \text{...(i)}$$

When Jogger is 39 m away, then $u = -39$ m

Using Eq. (i), we get

$$v = \frac{fu}{u-f} = \frac{1\ (-39)}{-39-1}$$

or $$v = \frac{39}{40} \text{ m}$$

As the Jogger is running at a constant speed of 5 m/s, after 1 s, the position of the image (v) for

$u = -39 + 5$

$\Rightarrow$ $u = -34$ m

Again, using Eq. (i), we get

$$\Rightarrow v = \frac{1(-34)}{-34-1}$$

$$\Rightarrow v = \frac{34}{35} \text{ m}$$

Difference in apparent position of Jogger in 1s

$$= \frac{39}{40} - \frac{34}{35}$$

$$= \frac{1365 - 1360}{1400} = \frac{1}{280} \text{ m}$$

Average speed of Jogger's image $= \frac{1}{280}$ m/s

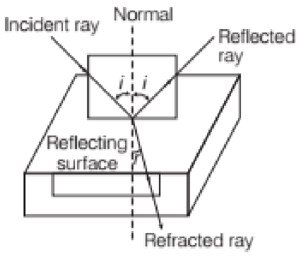
Similarly, for $u = -29$ m, -19 m and -9 m, average speed of Jogger image is $\frac{1}{150}$ m/s, $\frac{1}{60}$ m/s, $\frac{1}{10}$ m/s, respectively.

The speed increases as the Jogger approaches the car. This can be experienced by the person in the car.

|TOPIC 2|

Refraction

Reflection involves change in path of light without any change in the medium, whereas refraction involves change in the path of light due to change in the medium.



When a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other. The direction of propagation of an obliquely incident ray of light, that enters the other medium, changes at the interface of two media. This phenomenon is called **refraction** of light.

Laws of Refraction

(i) The incident ray, the refracted ray and the normal to the refracting surface (or interface) at the point of incidence, all lie in the same plane.

(ii) The ratio of the sine angle of incidence to the sine angle of refraction is constant for the two given media. This constant is denoted by ${}^a\mu_b$ and is called the **relative refractive index** of medium b with respect to medium a.

$$\therefore \quad \boxed{\frac{\sin i}{\sin r} = {}^a\mu_b}$$

This law is also called **Snell's law** of refraction.

Refractive Index

The refractive index or index of refraction μ of a material is the ratio of the speed of light (c) in vacuum to the speed of light in the medium (v).

Mathematically, refractive index is given by the relation

$$\mu = \frac{\text{Speed of light in the vacuum}}{\text{Speed of light in the material}} = \frac{c}{v}$$

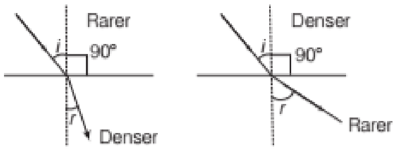
It is also referred as absolute refractive index of the substance.

Refractive Index of Some Substance Media

Substance medium	Refractive index
Ethyl alcohol	1.362
Water, H_2O	1.333
Air	1.000293
Oxygen, O_2	1.000271

Relative refractive index is a measure of how much light bends, when it travels from one medium to another medium.

If light travels from optical rarer medium to optical denser medium, then it bends towards the normal, i.e. $i > r$. On the other hand, if light travels from optical denser medium to optical rarer medium, then light bends away from the normal, i.e. $i < r$.



The medium in which the speed of light is higher with respect to other medium, is said to be optically **denser medium**. Optical density is the ratio of the speed of light in two media.

Optical density should not be confused with mass density, which is mass per unit volume. It is possible that, mass density of an optically denser medium may be less than that of an optically rarer medium. e.g. Turpentine and water. Mass density of turpentine is less than that of water but its optical density is higher.

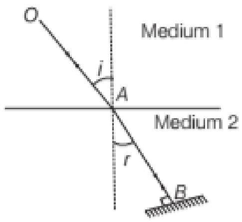
Principle of Reversibility of Light

When a light ray, after suffering any number of reflections and refractions, has its final path reversed, it travels back along its entire initial path. This is called **principle of reversibility of light**. In the figure, OA is an incident ray in medium 1 and AB is the refracted ray in medium 2. By Snell's law, the refractive index of medium 2 relative to medium 1 is given by

$${}^1\mu_2 = \frac{\sin i}{\sin r} \qquad \text{...(i)}$$

where, i and r are the angles of incidence and refraction, respectively.

Suppose the ray AB is reflected back by a plane mirror. Now, BA is the incident ray and AO is the refracted ray.



Correspondingly, r is angle of incidence and i is angle of refraction. Again, by Snell's law, the refractive index of medium 1 relative to medium 2 is given by

$$^{2}\mu_1 = \frac{\sin r}{\sin i} \quad \text{...(ii)}$$

Multiplying Eqs. (i) and (ii), we get

$$^{1}\mu_2 \times {}^{2}\mu_1 = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = 1$$

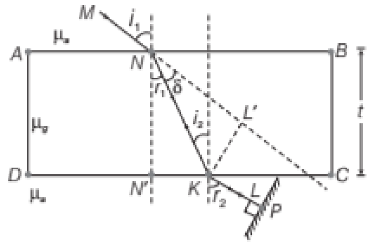
or $$\boxed{^{1}\mu_2 = \frac{1}{^{2}\mu_1}}$$

Thus, the refractive index of medium 2 relative to medium 1 is equal to the reciprocal of the refractive index of medium 1 relative to medium 2.

Refraction of Light Through a Rectangular Glass Slab

Let $ABCD$ be a rectangular glass slab. A ray of light is incident along MN on the face AB of the rectangular slab at $\angle i_1$. It is refracted along NK with $\angle r_1$.

The refracted ray NK falls on face CD with $\angle i_2$ and emerges out along KL with $\angle r_2$.



Applying Snell's law at N,

$$\mu_a \times \sin i_1 = \mu_g \times \sin r_1$$

or $$\frac{\sin i_1}{\sin r_1} = \frac{\mu_g}{\mu_a} = {}^{a}\mu_g \quad \text{....(i)}$$

Again, applying Snell's law at K,

$$\mu_g \times \sin i_2 = \mu_a \times \sin r_2$$

$$\Rightarrow \frac{\mu_a}{\mu_g} = \frac{\sin i_2}{\sin r_2} = {}^{g}\mu_a \quad \text{.... (ii)}$$

According to the principle of reversibility of light, when final path of a light ray after suffering a number of reflections and refractions is reversed, then the ray retraces its entire path.

Now, imagine a plane mirror P held normal to KL so that on reflection from mirror, path KL is reversed. The ray would retrace its entire path. For the reversed ray, the application of Snell's law at K gives

$$\mu_a \times \sin r_2 = \mu_g \times \sin i_2$$

$$\frac{\mu_g}{\mu_a} = \frac{\sin r_2}{\sin i_2} = {}^{a}\mu_g \quad \text{...(iii)}$$

Multiplying Eqs. (ii) and (iii), we get

$$\frac{\sin i_2}{\sin r_2} \times \frac{\sin r_2}{\sin i_2} = {}^{g}\mu_a \times {}^{a}\mu_g$$

$$1 = {}^{g}\mu_a \times {}^{a}\mu_g, \; {}^{a}\mu_g = \frac{1}{^{g}\mu_a}$$

From Eqs. (i) and (iii), we get

$$\frac{\sin i_1}{\sin r_1} = \frac{\sin r_2}{\sin i_2} \quad \text{...(iv)}$$

As, $i_2 = r_1$ [alternate angles]

$\therefore$ $\sin i_2 = \sin r_1$

From Eq. (iv), we get

$$\sin r_2 = \sin i_1 \text{ or } r_2 = i_1$$

Hence, the emergent ray KL is parallel to the incident ray MN as shown in the figure. We observe that the incident ray MN is displaced laterally, on suffering two refractions through a glass slab.

Expression for Lateral Displacement

Now, from K, draw $KL' \perp MN$ produced.

$\therefore$ Lateral displacement of the ray on passing through the parallel slab = KL'.

Let $\angle KNL' = \delta$ = deviation on first refraction.

In $\Delta NKL'$, $\sin \delta = \dfrac{KL'}{NK}$

$$\therefore \quad KL' = NK \sin \delta \quad \text{...(v)}$$

In $\Delta NN'K$, $\cos r_1 = \dfrac{NN'}{NK}$

$$\therefore \quad NK = \frac{NN'}{\cos r_1} = \frac{t}{\cos r_1}$$

where, $t = NN'$ = thickness of glass slab.

From Eq. (v), we get, $KL' = \dfrac{t}{\cos r_1} \sin \delta$

or $$\boxed{KL' = \frac{t \sin(i_1 - r_1)}{\cos r_1}} \quad \text{...(vi)}$$

This is the required expression for lateral displacement (or shift), which is obviously proportional to thickness (t) of glass slab. Further, lateral displacement (or shift) will increase with increasing angle of incidence (i_1).

EXAMPLE |1| A ray of light is incident at an angle of 60° on one face of a rectangular glass slab of thickness 0.1 m and refractive index 1.5. Calculate the lateral shift produced.

Sol. Given, angle of incidence, $i_1 = 60°$

Thickness of glass slab, $t = 0.1$ m

Refractive index, $\mu = 1.5$

Since, $\frac{\sin i_1}{\sin r_1} = \mu$

$\therefore \quad \sin r_1 = \frac{\sin i_1}{\mu} = \frac{\sin 60°}{1.5} = 0.5773$

$\left[\because \sin 60° = \frac{\sqrt{3}}{2} \text{ and } \sqrt{3} = 1.732\right]$

$r_1 = \sin^{-1}(0.5773) = 35.3°$

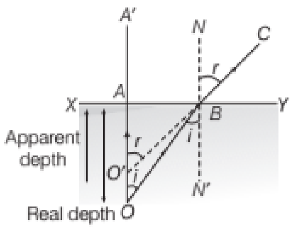
$\therefore$ Lateral shift $= \frac{t \sin(i_1 - r_1)}{\cos r_1}$

$= \frac{0.1 \sin(60° - 35.3°)}{\cos 35.3°} = \frac{0.1 \sin 24.7°}{\cos 35.3°}$

$= \frac{0.1 \times 0.418}{0.816} = 0.0513$ m

Apparent Depth and Normal Shift

The depth of an object immersed in water appears to be lesser than its actual depth. Let O be a point object at an actual depth OA below the free surface of water XY.



A ray of light incident normally on XY, along OA passes straight along OAA'. Another ray of light from O incident at $\angle i$ on surface XY along OB deviates away from normal. It is refracted at $\angle r$ along BC. On producing backwards BC meets OA at O'. Therefore, O' is virtual image of O.

Apparent depth $= AO'$

Real depth $= OA$

Clearly, $AO' < OA$

Now, $\angle BOA = \angle OBN' = i$ [alternate angles]

$\angle AO'B = \angle CBN = r$ [corresponding angles]

In ΔOAB, $\sin i = \frac{AB}{OB}$

In $\Delta O'AB$, $\sin r = \frac{AB}{O'B}$

As, light ray is travelling from denser medium to rarer medium.

$\therefore \quad {}^a\mu_w = \frac{\sin r}{\sin i}$

or $\quad {}^a\mu_w = \frac{AB}{O'B} \times \frac{OB}{AB} = \frac{OB}{O'B}$

B is close to A (as angles are very small). So, $OA \approx OB$ and $O'A \approx O'B$

$\therefore \quad \boxed{{}^a\mu_w = \frac{OA}{O'A} = \frac{\text{Real depth}}{\text{Apparent depth}}}$

If x is the real depth of water surface and ${}^a\mu_w$ is the refractive index of water with respect to air, then the normal shift (d) in position of point object is given by

d = Real depth − Apparent depth

$\therefore \quad d = x - \frac{x}{{}^a\mu_w}$

$\left[\because \text{apparent depth} = \frac{\text{real depth}}{{}^a\mu_w} = \frac{x}{{}^a\mu_w}\right]$

or $\quad \boxed{d = x\left(1 - \frac{1}{{}^a\mu_w}\right)}$

EXAMPLE |2| Velocity of light in glass is 2×10^8 m/s and that in air is 3×10^8 m/s. By how much would an ink dot appear to be raised, when covered by a glass plate 6 cm thick?

Sol. Given, velocity of light in glass, $v = 2 \times 10^8$ m/s

Velocity of light in air, $c = 3 \times 10^8$ m/s

$\therefore$ Refractive index of glass with respect to air,

${}^a\mu_g = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$

$\therefore$ Normal shift in the position of ink dot,

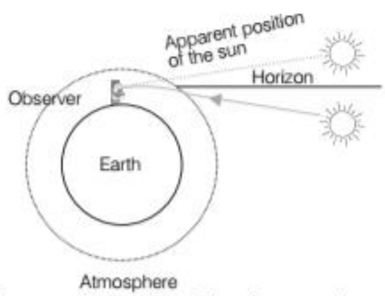
$d = t\left(1 - \frac{1}{{}^a\mu_g}\right)$ $[\because t = 6 \text{ cm}]$

$= 6\left(1 - \frac{1}{1.5}\right) = \frac{6 \times 0.5}{1.5} = 2$ cm

Effect of Atmospheric Refraction at Sunrise and Sunset

The density of atmosphere around the earth is not uniform throughout due to which, it has layers of different densities. The refraction of light due to variation in optical density of atmospheric layers is called **atmospheric refraction**.

Due to refraction of sunlight from atmosphere, the sun is visible a little before the actual sunrise and a little after the actual sunset.

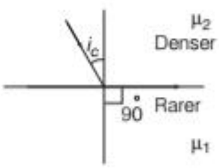


Advance sunrise and delayed sunset due to atmospheric refraction

The refractive index of air with respect to vacuum is 1.00029. Due to this, the apparent shift in the direction of the sun is about half a degree and the corresponding time difference between actual sunset and apparent sunset is about 2 min. The apparent flattening (oval shape) of the sun at sunset and sunrise is also due to atmospheric refraction.

Critical Angle

Critical angle for a pair of given media in contact can be defined as, "the angle of incidence in denser medium for which angle of refraction in rarer medium is 90°". The value of critical angle depends on the nature of two media in contact.



From Snell's law, $\mu_2 \times \sin i_c = \mu_1 \times \sin 90°$

$$\therefore \quad \frac{\mu_1}{\mu_2} = \frac{\sin i_c}{\sin 90°} \Rightarrow \frac{\mu_1}{\mu_2} = \sin i_c \qquad [\because \sin 90° = 1]$$

$$\text{or} \quad \frac{\mu_2}{\mu_1} = \frac{1}{\sin i_c} \Rightarrow {}^1\mu_2 = \frac{1}{\sin i_c}$$

Critical Angle of Some Transparent Media

Substance medium	Refractive index	Critical angle
Water	1.33	48.75°
Crown glass	1.52	41.14°
Dense flint glass	1.62	37.31°
Diamond	2.42	24.41°

EXAMPLE |3| If a ray of light travelling in air is incident on a glass surface with an angle of incidence 40°, it deviates through 15°, determine the critical angle for a glass-air interface.

Sol. Given, angle of incidence, $i = 40°$

Angle of deviation, $\delta = 15°$

Since, ray deviates towards the normal, therefore

$$r = i - \delta = 40° - 15° = 25°$$

As we know that,

$$\mu = \frac{\sin i}{\sin r} = \frac{1}{\sin i_c}$$

$$\Rightarrow \sin i_c = \frac{\sin r}{\sin i} = \frac{\sin 25°}{\sin 40°} = \frac{0.4226}{0.6428}$$

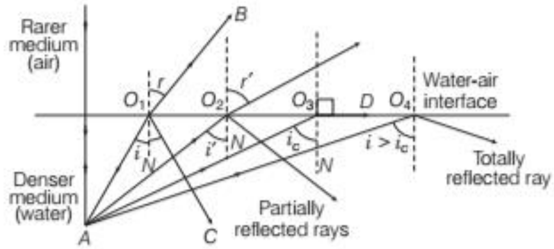
$$\Rightarrow \sin i_c = 0.6574$$

$$\therefore \quad i_c = \sin^{-1}(0.6574) = 41.1°$$

$$\Rightarrow \quad i_c = 41.1°$$

TOTAL INTERNAL REFLECTION (TIR)

When a ray of light travelling from denser medium to rarer medium, is incident at the interface of two media at an angle greater than the critical angle for the two media, the ray is totally reflected back to denser medium, this phenomena is called **Total Internal Reflection** (TIR).



Refraction and internal reflection of rays from a point *A* in the denser medium (water) incident at different angles at the interface with a rarer medium (air)

Necessary conditions for total internal reflection to take place are as follows

(i) The ray incident on the interface of two media should travel in the denser medium.

(ii) The angle of incidence should be greater than critical angle for the two media.

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. Which of the following quantity remains unchanged after refraction?

(a) Speed of light (b) Intensity of light
(c) Wavelength of light (d) Frequency of light

2. A ray of light strikes an air-glass interface at an angle of incidence ($i = 60°$) and gets refracted at an angle of refraction r. On increasing the angle of incidence ($i > 60°$), the angle of refraction r

(a) decreases (b) remains same

(c) is equal to 60° (d) increases

3. A ray of light strikes a transparent rectangular slab of refractive index $\sqrt{2}$ at an angle of incidence of 45°. The angle between the reflected and refracted ray is

(a) 75° (b) 90° (c) 105° (d) 120°

4. Speed of light in air is 3.0×10^8 m/s. Speed of light in the glass of refractive index 1.5 will be

(a) 1.5×10^8 m/s (b) 2.0×10^8 m/s

(c) 1.8×10^8 m/s (d) 2.5×10^8 m/s

5. The refractive indices of water and glass with respect to air are 4/3 and 5/3, respectively. The refractive index of glass with respect to water will be

(a) 1/3 (b) 4/3 (c) 5/4 (d) 20/9

6. If the value of critical angle is 30° for total internal reflection from any medium to vacuum, then speed of light in that medium

(a) 3×10^8 m/s (b) 1.5×10^8 m/s

(c) 6×10^8 m/s (d) 4.5×10^8 m/s

7. If in denser medium, incidence angle is equal to critical angle, then refraction angle will be

(a) 0° (b) 45°

(c) 90° (d) 180°

8. The ratio $\frac{\text{real depth}}{\text{apparent depth}}$ is equal to

(a) refractive index of denser medium with respect to air

(b) refractive index of denser medium with respect to rare medium

(c) refractive index of rare medium with respect to air

(d) refractive index of rare medium with respect to denser medium

9. The phenomena involved in the reflection of radiowaves by ionosphere is similar to **NCERT Exemplar**

(a) reflection of light by a plane mirror

(b) total internal reflection of light in air during a mirage

(c) dispersion of light by water molecules during the formation of a rainbow

(d) scattering of light by the particles of air

VERY SHORT ANSWER Type Questions

10. When monochromatic light travels from one medium to another, its wavelength changes, but frequency remains same. Explain. **Delhi 2011**

11. For the same value of angle of incidence, the angles of refraction in three media A, B and C are 15°, 25° and 35°, respectively. In which medium would the velocity of light be minimum? **All India 2012**

12. A ray of light strikes on air-glass interface at an angle of incidence ($< i = 60°$) and gets refracted at an angle of refraction ($< r$). What will happen to the angle of refraction on increasing the angle of incidence?

13. Why does a crack in a glass window pane appear silvery?

14. The refractive index of diamond is much higher than that of glass. How does a diamond cutter make use of this fact? **All India 2011**

15. Why prisms are used in many optical instruments?

16. Which of the two main parts of an optical fibre has a higher value of refractive index?

SHORT ANSWER Type Questions

17. When monochromatic light travels from a rarer to a denser medium, explain the following, giving reasons.

(i) Is the frequency of reflected and refracted light same as the frequency of incident light?

(ii) Does the decrease in speed imply a reduction in the energy carried by light wave? **Delhi 2013**

18. Mention any two situations in which Snell's law of refraction fails.

19. A ray of light is incident at a glass-water interface at an angle of i, it emerges finally parallel to the surface water, then what will be the value of μ_g?

20. Why does the sun rising in the sky appear oval in shape?

21. Choose the statement as wrong or right and justify.

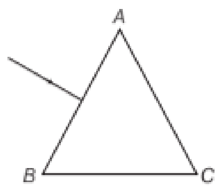
(i) Snell's law is verified for all types of surface.

(ii) Total internal reflection only takes place, when light travels from rarer to denser medium.

22. (i) Write the necessary conditions for the phenomenon of total internal reflection to occur.

(ii) Write the relation between refractive index and critical angle for a given pair of optical media. **Delhi 2013**

23. The figure shows a ray of light falling normally on the face AB of an equilateral glass prism having refractive index 3/2, placed in water of refractive index 4/3. Will this ray suffer total internal reflection on striking the face AC? Justify your answer. **CBSE 2018**



LONG ANSWER Type I Questions

24. Define the following with required formula.

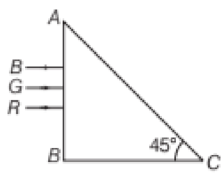
(i) Apparent depth

(ii) Lateral displacement (or shift)

(iii) Critical angle

25. A beaker contains water upto height h_1 and kerosene of height h_2 above water surface, so that the total height of (water + kerosene) is $h_1 + h_2$. Refractive index of water is μ_1 and that of kerosene is μ_2. What will be the apparent shift in position of the bottom of the beaker as viewed from above?

26. Three light rays, red (R), green (G) and blue (B) are incident on a right angled prism ABC at face AB. The refractive indices of the material of the prism for red, green and blue wavelengths are 1.39, 1.44 and 1.47, respectively. Out of the three, which colour of ray will emerge out of face AC? Justify your answer. Trace the path of these rays after passing through face AB.



27. Show that for a material with refractive index $\mu \geq \sqrt{2}$, light incident at any angle shall be guided along a length perpendicular to the incident face. **NCERT Exemplar**

28. Three immiscible liquids of densities $d_1 > d_2 > d_3$ and refractive indices $\mu_1 > \mu_2 > \mu_3$ are put in a beaker. The height of each liquid column is $\frac{h}{3}$. A dot is made at a bottom of the beaker. For near normal vision, find the apparent depth of the dot. **NCERT Exemplar**

> **Hints:** The image formed by first medium acts as an object for second medium.

LONG ANSWER Type II Question

29. Explain the phenomenon of total internal reflection. Describe how TIR takes place in optical fibre. State any two uses of it.

NUMERICAL PROBLEMS

30. What is the ratio of the velocities of two light waves travelling in vacuum and having wavelengths 4000 Å and 8000 Å?

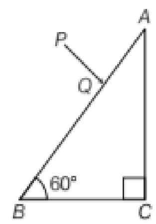
31. What is the critical angle for a material of refractive index $\sqrt{2}$?

32. Determine the lateral displacement of the ray of light passing through a 15 cm thick glass slab with opposite sides parallel, if the angle of incidence of the ray is 60°. Given, $n = 1.5$.

33. A ray of light is incident at an angle of 45° on one face of a rectangular glass slab of thickness 10 cm and refractive index 1.5. Calculate the lateral shift produced.

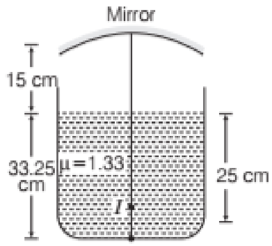
34. What is the apparent position of an object below a rectangular block of glass 6 cm thick, if a layer of water 4 cm thick is on the top of the glass? Given, $n_{ga} = 1.5$ and $n_{wa} = 1.33$.

35. A ray PQ incident normally on the refracting face BA is refracted in the prism BAC made of material of refractive index 1.5. Complete the path of ray through the prism. From which face will the ray emerge? Justify your answer. **All India 2016**



36. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 upto the same height, by what distance would the microscope have to be moved to focus on the needle again? **NCERT**

37. A container is filled with water ($\mu = 1.33$) upto a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. What will be the focal length?]



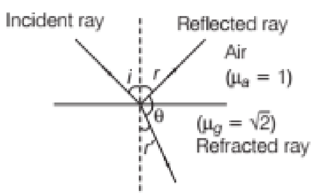
HINTS AND SOLUTIONS

1. (d) Refraction does not change the frequency of light.
2. (d) From Snell's law of refraction,

$${}^a\mu_g = \frac{\sin i}{\sin r} = \text{constant} \quad ...(i)$$

Since, angle of incidence increase, the angle of refraction has to increase. So, that the ratio $\left(\frac{\sin i}{\sin r}\right)$ is a constant according to Eq. (i).

3. (c) Given, $i = 45°$



From Snell's law, $\frac{\sin i}{\sin r'} = \frac{\mu_g}{\mu_a}$

$$\Rightarrow \quad \frac{\sin 45°}{\sin r'} = \frac{\sqrt{2}}{1}, \ \sin r' = \frac{1}{2} \Rightarrow r' = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

From diagram, $r + \theta + r' = 180°$

$$i + \theta + 30° = 180° \quad (\because i = r)$$

$$45 + \theta + 30° = 180°$$

$$\theta = 180° - 75° = 105°$$

Hence, the angle between reflected and refracted ray is 105°.

4. (b) Refractive index of glass

$$= \frac{\text{Speed of light in air } (3\times 10^8)}{\text{Speed of light in glass } (x)}$$

$$\Rightarrow \quad x = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/s}$$

5. (c) Given, ${}_a n_w = \frac{4}{3}$, ${}_a n_g = \frac{5}{3}$

$$\therefore \quad {}_a n_w \times {}_w n_g = {}_a n_g$$

$${}_w n_g = \frac{{}_a n_g}{{}_a n_w} \frac{5/3}{4/3} = \frac{5}{3}\times\frac{3}{4} = \frac{5}{4}$$

6. (b) From Snell's law,

$$\sin C = {}_1 n_2 = \frac{v_1}{v_2}$$

where, C = critical angle = 30°

v_1 and v_2 are speed of light in medium and vacuum, respectively.

We know that, $v_2 = 3\times10^8$ m/s

$$\therefore \quad \sin 30° = \frac{v_1}{3\times10^8}$$

$$\Rightarrow v_1 = 3\times10^8\times\frac{1}{2} \Rightarrow v_1 = 1.5\times10^8 \text{ m/s}$$

7. (c) If incidence angle, i = critical angle C, then refraction angle, $r = 90°$.
8. (b) As we know, refractive index of denser medium w.r.t. rare medium $= \frac{\text{Real depth}}{\text{Apparent depth}}$
9. (*b*) The phenomenon involved in the reflection of radiowaves by ionosphere is similar to total internal reflection of light in air during a mirage i.e., angle of incidence is greater than critical angle.
10. Because refractive index for a given pair of media depends on the ratio of wavelengths and velocity of light in two media but not on frequency. So, frequency remains constant during refraction of light.
11. From Snell's law, $\mu = \frac{\sin i}{\sin r} = \frac{c}{v}$

$\Rightarrow v \propto \sin r$, for given value of i.

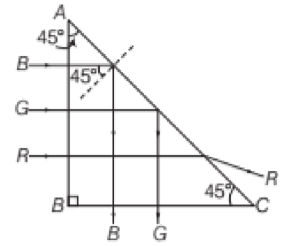
Smaller the angle of refraction, smaller the velocity of light in medium.

Velocity of light is minimum in medium A as the angle of refraction is minimum, i.e. 15°.

12. From Snell's law, $\mu = \frac{\sin i}{\sin r}$ = constant

Since, angle of incidence increases, the angle of refraction has to increase, so that the ratio remains constant.

13. Whenever rays of light travels through glass, they strike the glass-air interface at an angle greater than critical angle of glass. They are totally reflected, hence crack appears silvery.
14. The refractive index of diamond is much higher than that of glass. Due to high refractive index, the critical angle for diamond-air interface is low. The diamond is cut suitably, so that the light entering the diamond from any face suffers multiple total internal reflections at the various surfaces. This gives sparkling effect to the diamonds.



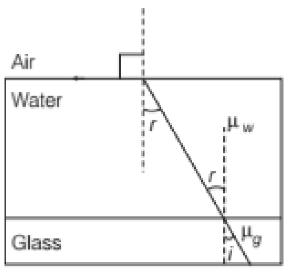
15. Since, prisms can bend the light rays by 90° and 180° by total internal reflection, so they are used in many optical instruments.

16. There are two main parts of the optical fibre
(i) Core
(ii) Cladding
The refractive index of core is greater than that of cladding such that TIR can occurs.

17. (i) The frequency of reflected and refracted light remains same as that of incident light because frequency only depends on the source of light.
(ii) Since, the frequency remains same, hence there is no reduction in energy.

18. Snell's law of refraction fails in two situations
(i) When TIR (total internal reflection) takes place at angle greater than the critical angle.
(ii) When light is incident normally on a surface, as $i = 0$, $r = 0$.

19. For glass-water interface, applying Snell's law,



$$\frac{\sin i}{\sin r} = \frac{\mu_w}{\mu_g} \Rightarrow \mu_g = \left(\frac{\mu_w \sin r}{\sin i}\right) \qquad \text{...(i)}$$

For water-air interface,

$$\frac{\sin r}{\sin 90^\circ} = \frac{1}{\mu_w} \Rightarrow \sin r = \frac{1}{\mu_w} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\mu_g = \frac{\mu_w \times \frac{1}{\mu_w}}{\sin i} \Rightarrow \mu_g = \frac{1}{\sin i}$$

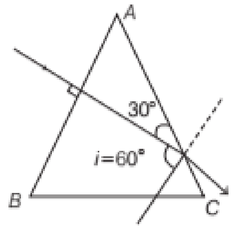
20. It is due to the refraction of sunlight as it travels through the earth's atmosphere. Refraction of light by these layers can make the sun appear flattened or distorted. Objects closer to the horizon are raised upwards most and the lower limb of the sun is raised more than the top making it appear oval.

21. (i) Refer to text on pages 360 and 361.
(ii) Refer to text on page 363.

22. (i) Refer to text on page 363.
(ii) Refer to text on page 363.

23. Given, refractive index of water, $\mu_w = 4/3$

Refractive index of glass prism, $\mu_g = \frac{3}{2}$



For total internal reflection occurrence the incident angle must be greater than critical angle.

∴ Let us calculate critical angle C.

As we know that, $\sin C = \frac{1}{\mu}$

where, $\mu = \frac{\text{refractive index of glass } ({}_a\mu_g)}{\text{refractive index of water } ({}_a\mu_w)}$

$$\therefore \quad \sin C = \frac{1}{\left(\frac{{}_a\mu_g}{{}_a\mu_w}\right)} = \frac{1}{\left(\frac{3/2}{4/3}\right)} = \frac{1}{9/8}$$

or $\quad \sin C = \frac{8}{9} = 0.88 \Rightarrow C = 61.6^\circ$

[As $\sin 60^\circ = \sqrt{3}/2 = 0.86$]

As the critical angle, i.e, 61.6° is greater than the angle of incidence, i.e. 60°, hence TIR will not occurs.

24. (i) Refer to text on page 362.
(ii) Refer to text on page 361.
(iii) Refer to text on page 363.

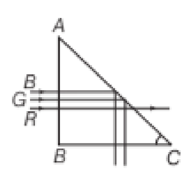
25. ∴ Apparent depth, $d = d_1 + d_2 = \left(1 - \frac{1}{\mu_1}\right)h_1 + \left(1 - \frac{1}{\mu_2}\right)h_2$

26. By geometry, angle of incidence (i) at face AC for all three rays is 45°. Light suffers total internal reflection for which this angle of incidence is greater than critical angle.

$$i > i_c \Rightarrow \sin i > \sin i_c \text{ or } \sin 45^\circ > \sin i_c$$

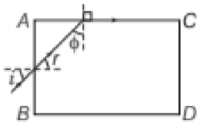
$$\Rightarrow \frac{1}{\sin 45^\circ} < \frac{1}{\sin i_c} \Rightarrow \sqrt{2} < \mu$$

Total internal reflection takes place on AC for rays with $\mu > \sqrt{2} = 1.414$, i.e. green and blue colour suffer total internal reflection, whereas red undergoes refraction.



27. Any ray entering at an angle i shall be guided along AC, if the ray makes an angle ϕ with the face AC greater than the critical angle as per the principle of total internal reflection, $\phi + r = 90°$, therefore $\sin \phi = \cos r$.

$$\Rightarrow \quad \sin \phi \geq \frac{1}{\mu} \quad \Rightarrow \quad \cos r \geq \frac{1}{\mu}$$



or $1 - \cos^2 r \leq 1 - \frac{1}{\mu^2} \Rightarrow \sin^2 r \leq 1 - \frac{1}{\mu^2}$ $\left[\because 1 - \cos^2 r = \sin^2 r\right]$

Since, $\sin i = \mu \sin r$

$$\frac{1}{\mu^2}\sin^2 i \leq 1 - \frac{1}{\mu^2}$$

or $$\sin^2 i \leq \mu^2 - 1$$

When $i = \frac{\pi}{2}$, then we have smallest angle ϕ.

If the angle ϕ is greater than the critical angle, then all other angles of incidence shall be more than the critical angle.

Thus, $\quad 1 \leq \mu^2 - 1$ or $\mu^2 \geq 2$

$$\Rightarrow \quad \mu \geq \sqrt{2}$$

This is the required result.

28. Let the apparent depth be O_1 for the object seen from m_2, then $O_1 = \frac{\mu_2}{\mu_1} \cdot \frac{h}{3}$.

Since, apparent depth = real depth/refractive index (μ).

Since, the image formed by medium 1 acts as an object for medium 2.If seen from μ_3, the apparent depth is O_2.

Similarly, the image formed by medium 2 acts as an object for medium 3.

$$O_2 = \frac{\mu_3}{\mu_2}\left(\frac{h}{3} + O_1\right)$$

$$= \frac{\mu_3}{\mu_2}\left(\frac{h}{3} + \frac{\mu_2 h}{\mu_1 3}\right) = \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)$$

As, seen from outside, the apparent height is

$$O_3 = \frac{1}{\mu_3}\left(\frac{h}{3} + O_2\right) = \frac{1}{\mu_3}\left[\frac{h}{3} + \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)\right]$$

$$= \frac{h}{3}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right)$$

This is the required expression of apparent depth.

29. Refer to text on pages 363.

30. Since, light travels in vacuum with a constant velocity, i.e. 3×10^8 m/s, hence ratio of velocities of all wavelengths remains same.

31. We know that, $\mu = \frac{1}{\sin C}$

$$\Rightarrow \quad \sin C = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad C = 45°$$

32. Using lateral shift, $d = \frac{t \sin(i_1 - r_1)}{\cos r_1}$

Refer to Example 1 on page 362.

33. Given, $i_1 = 45°$, $t = 10$ cm $= 0.1$ m, $\mu = 1.5$

Lateral shift = ?

By Snell's law, $\mu = \frac{\sin i_1}{\sin r_1} \Rightarrow \sin r_1 = \frac{\sin i_1}{\mu} = \frac{\sin 45°}{1.5}$

$$\Rightarrow \quad \sin r_1 = \frac{0.707}{1.5} \qquad \left[\therefore \sin 45° = 1/\sqrt{2}, \sqrt{2} = 1.414\right]$$

$$\Rightarrow \quad \sin r_1 = 0.4713$$

$$\Rightarrow \quad r_1 = \sin^{-1}(0.4713) \Rightarrow r_1 = 28.12°$$

$$\text{Lateral shift} = \frac{t \sin(i_1 - r_1)}{\cos r_1} = \frac{0.1 \sin(45° - 28.12°)}{\cos 28.12°}$$

$$= \frac{0.1 \sin 16.88°}{\cos 28.12°} = \frac{0.1 \times 0.2904}{0.8819} = 0.033 \text{ m}$$

34. Here, $\quad \mu = \frac{\text{real depth / thickness of object}}{\text{apparent depth}}$

Now, due to refraction at two different boundaries, the apparent depth of object is

$$\text{apparent depth} = \frac{\text{thickness of glass}}{\mu_{glass}} + \frac{\text{thickness of water}}{\mu_{water}}$$

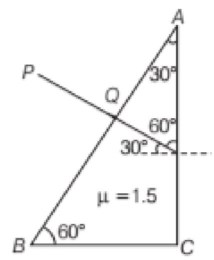
$$= \frac{6}{1.5} + \frac{4}{1.3} = 3 + 4 = 7 \text{ cm}$$

35. Given, refractive index of the material of the prism, $\mu = 1.5$

$\therefore$ Critical angle for the material,

$$\sin C = \frac{1}{\mu} = \frac{1}{1.5} = 2/3$$

$$\Rightarrow \quad C = \sin^{-1}\left(\frac{2}{3}\right) \doteq 42°.$$



From the ray diagram, it is clear that angle of incidence $i = 30° < C$.

Therefore, the ray incident at the face AC will not suffer total internal reflection and merges out through this face.

36. ***Case* I** When tank is filled with the water.

Given, the apparent depth = 9.4 cm

Height of water, t = 12.5 cm

So, real depth = 12.5 cm

Refractive index of water,

$$\mu_w = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{12.5}{9.4} = 1.33$$

***Case* II** When tank is filled with the liquid.

Refractive index of liquid, $\mu_l = 1.63$

Again, $\mu_l = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

$\Rightarrow \quad 1.63 = \dfrac{12.5}{\text{Apparent depth}}$

Apparent depth $= \dfrac{12.5}{1.63} = 7.67$ cm

$\therefore$ The microscope is shifted by 9.4 − 7.67 = 1.73 cm.

37. Distance of object from mirror

$$= 15 + \frac{33.25}{4} \times 3 = 39.93 \text{ cm}$$

Distance of image from the mirror

$$= 15 + \frac{25}{4} \times 3$$

$$= 33.75 \text{ cm}$$

Using mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\Rightarrow \quad \frac{1}{-33.75} - \frac{1}{39.93} = \frac{1}{f}$$

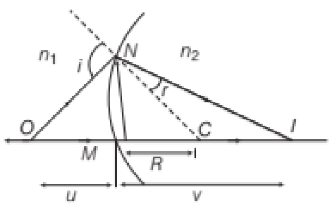
$$\therefore \quad f = -18.3 \text{ cm}$$

|TOPIC 3|

Refraction at Spherical Surfaces and Lenses

REFRACTION AT A SPHERICAL SURFACE

A refracting surface which forms a part of a sphere of transparent refracting material is called a **spherical refracting surface**.



Refraction at a spherical surface

In the figure, the geometry of formation of image I of an object O and the principal axis of a spherical surface with centre of curvature C and radius of curvature R.

Assumptions

(i) The aperture of the surface is small as compared to other distances involved.

(ii) NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\tan NOM = \frac{MN}{OM}, \quad \tan NCM = \frac{MN}{MC},$$

$$\tan NIM = \frac{MN}{MI}$$

For small angles, $\tan\theta \approx \sin\theta \approx \theta$

So, $\angle NOM = \dfrac{MN}{OM}$

$$\angle NCM = \frac{MN}{MC}$$

$$\angle NIM = \frac{MN}{MI}$$

For ΔNOC, i is the exterior angle.

$$\therefore \quad i = \angle NOM + \angle NCM = \frac{MN}{OM} + \frac{MN}{MC} \qquad \ldots\text{(i)}$$

For ΔNIC, $\angle NCM$ is the exterior angle.

$$\therefore \quad \angle NCM = r + \angle NIM$$

or $\quad r = \angle NCM - \angle NIM$

i.e. $$r = \frac{MN}{MC} - \frac{MN}{MI} \qquad \ldots\text{(ii)}$$

By Snell's law, $n_1 \sin i = n_2 \sin r$

For small angles, $\quad n_1 i = n_2 r$

Substituting the values of i and r from Eqs. (i) and (ii), we get

$$n_1\left(\frac{MN}{OM} + \frac{MN}{MC}\right) = n_2\left(\frac{MN}{MC} - \frac{MN}{MI}\right)$$

or $$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \qquad \ldots\text{(iii)}$$

Applying new Cartesian sign conventions,

$$OM = -u, \quad MI = +v$$

$$MC = +R$$

Substituting these values in Eq. (iii), we get

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

This equation holds for any curved spherical surface.

EXAMPLE |1| Light from a point source in air falls on a spherical glass surface ($n = 1.5$ and radius of curvature $= 20$ cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

Sol. Given, object distance, $u = -100$ cm,

$R = +20$ cm, $n_1 = 1$, $n_2 = 1.5$, image distance, $v = ?$

We know that, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$$\Rightarrow \quad \frac{1.5}{v} + \frac{1}{100} = \frac{1.5 - 1}{20}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{20} - \frac{1}{100}$$

$$= \frac{2.5 - 1}{100} = \frac{1.5}{100}$$

$$v = +100 \text{ cm}$$

Thus, the image is formed at a distance of 100 cm from the glass surface in the direction of incident light.

Cartesian Sign Convention for Spherical Surfaces

(i) The principal axis of the spherical surface is taken as X-axis and the optical centre as origin. Here, the principal axis is the diameter extended.

(ii) The direction of the incident light is taken as the positive direction of X-axis and opposite to it is taken as negative.

(iii) The upward direction is taken as positive and the downward direction as negative.

LENS

Lens is a transparent medium bounded by two surfaces of which one or both surfaces are spherical.

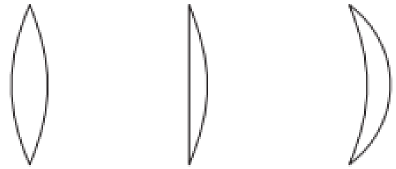
Lenses are of two types

(i) Convex or converging lens

(ii) Concave or diverging lens

Convex or Converging Lens

A lens which is thicker at the centre and thinner at its ends is called convex lens. Convex lenses are of three types as shown below.

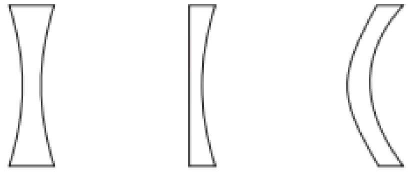


(a) Double convex lens (b) Plano-convex lens (c) Concavo-convex lens

Note A convex lens is also known as converging lens because it converges a parallel beam of light rays passing through it. A double convex lens is simply called convex lens.

Concave or Diverging Lens

A lens which is thinner at the centre and thicker at its ends is called a concave lens. Concave lenses are of three types as shown below.

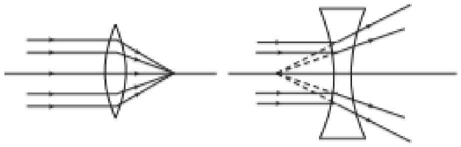


(a) Double concave lens (b) Plano-concave lens (c) Convexo-concave lens

Note A concave lens is also known as diverging lens because it diverges a parallel beam of light rays passing through it. A double concave lens is simply called concave lens.

Converging and Diverging Action of Lenses

As convex lens converges all the light rays, coming parallel to its principal axis at a point, it is also called converging lens. Concave lens diverges all the light rays coming parallel to its principal axis. So, it is also called diverging lens.

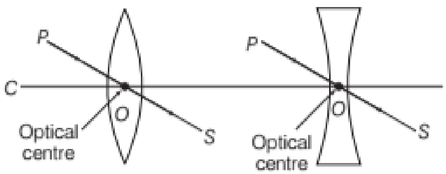


Converging lens Diverging lens

The converging and diverging action of lens can be explained by considering a lens made up of large number of different small angle prisms. In a convex lens, the base of prism is towards principal axis and in concave lens, base of prism is away from the principal axis.

Some Definitions Related to Lenses

(i) **Optical centre** The optical centre is a point lying on the principal axis of the lens, directed to which incident rays pass without any deviation in the path, i.e. the centre point of a lens is known as its optical centre.



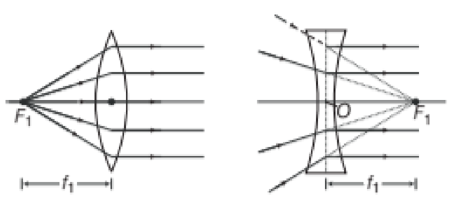
(ii) **Centre of curvature** The centres of the two imaginary spheres of which the lens is a part, are called centres of curvature of the lens. A lens has two centres of curvature with respect to its two curved surfaces.

(iii) **Radii of curvature** The radii of the two imaginary spheres of which the lens is a part are called radii of curvature of the lens. A lens has two radii of curvature. These may or may not be equal.

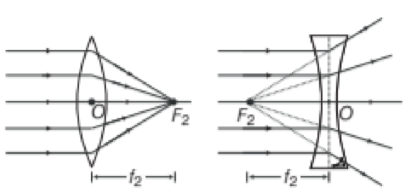
(iv) **Principal axis** The imaginary line joining the two centres of curvature is called principal axis of lens. Principal axis also passes through the optical centre.

(v) **Principal focus** Lens has two principal foci.

(a) **First principal focus** It is a point on the principal axis of lens, the rays starting from this point in convex lens or rays directed to this point in concave lens become parallel to principal axis after refraction.



(b) **Second principal focus** It is a point on the principal axis at which the rays coming parallel to the principal axis converge (convex lens) or passing through it appear to diverge (concave lens) at this point after refraction from the lens.



Both the foci of convex lens are real, while that of concave lens are virtual.

(vi) **Aperture** The effective diameter of the circular outline of a spherical lens is called its aperture.

(vii) **Refractive axis** It is an imaginary axis at the optical centre perpendicular to the principal axis which represents the lens.

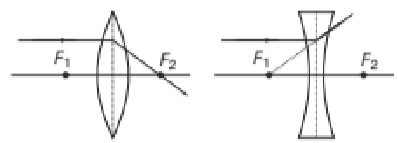
(a) Real path of ray (b) Path of ray as shown with reference to refractive axis

Note When the object is at infinity, the distance of image from the lens will be equal to the focal length of the lens.

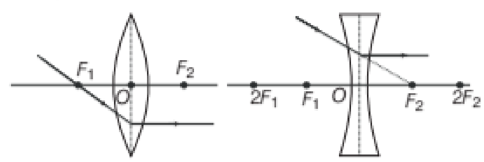
Image Formation in Lenses Using Ray Diagrams

We can represent image formation in lenses using ray diagrams. For drawing ray diagrams in lenses like spherical mirrors, we consider any two of the following rays.

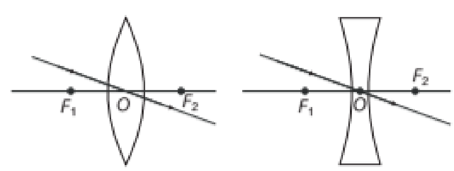
(i) Rays which are parallel to the principal axis after refraction, will pass through principal focus in case of convex lens and will appear to be coming from principal focus in case of concave lens.



(ii) Rays passing through or directed to the focus will emerge parallel to the principal axis.



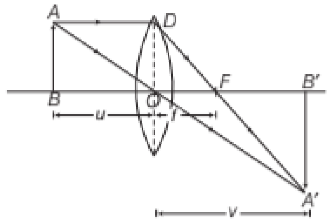
(iii) Rays directed to optical centre will emerge out undeviated.



THIN LENS FORMULA

It is a relation between focal length of a lens and distances of object and image from optical centre of the lens.

Let O be the optical centre and f be the principal focus of a convex lens of focal length $OF = f$. AB is an object held perpendicular to the principal axis of the lens at a distance beyond focal length of the lens. A real, inverted and magnified image $A'B'$ is formed as shown in the figure. As, $\Delta A'B'O$ and ΔABO are similar.



$$\therefore \qquad \frac{A'B'}{AB} = \frac{OB'}{OB} \qquad \text{...(i)}$$

Again, $\Delta\, A'B'F$ and ΔDOF are similar.

$$\therefore \qquad \frac{A'B'}{OD} = \frac{FB'}{OF}$$

But $\qquad OD = AB$

$$\therefore \qquad \frac{A'B'}{AB} = \frac{FB'}{OF} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{OB'}{OB} = \frac{FB'}{OF} = \frac{OB' - OF}{OF}$$

Using new cartesian sign conventions,

Let $\qquad OB = -u, \quad OB' = +v,$

$$OF = +f$$

$$\therefore \qquad \frac{v}{-u} = \frac{v-f}{f}$$

$$\Rightarrow \qquad vf = -uv + uf$$

or $\qquad uv = uf - vf$

Dividing both sides by uvf, we get

$$\frac{uv}{uvf} = \frac{uf}{uvf} - \frac{vf}{uvf} \quad \Rightarrow \quad \boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

This is the thin lens formula.

This formula can also be proved for concave lens and for virtual images in the same way.

EXAMPLE |2| A convergent beam of light passes through the diverging lens of focal length 0.2 m and comes to focus 0.3 m behind the lens. Find the position of the point at which the beam would converge in the absence of the lens.

Sol. Given, focal length, $f = 0.2$ m

Image distance, $v = -0.3$ m

Object distance, $u = ?$

From thin lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

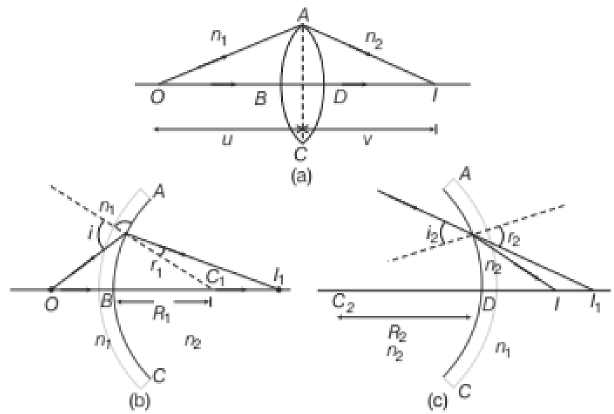
$$\Rightarrow \qquad \frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-0.3} - \frac{1}{0.2} = \frac{-0.5}{0.06}$$

$$\Rightarrow \qquad u = \frac{-0.06}{0.5}$$

$\therefore$ Object distance, $u = -0.12$ m

Refraction by a Lens : Lens Maker's Formula



The above figures show the image formation by a convex lens.

Assumptions Made in the Derivation

Some assumptions made from the derivation are as

(i) The lens is thin, so that distances measured from the poles of its surfaces can be taken as equal to the distance from the optical centre of the lens.

(ii) The aperture of the lens is small.

(iii) The object considered as a point lying on the principal axis of the lens.

(iv) The incident ray and refracted ray make small angles with the principal axis of the lens.

(v) A convex lens is made up of two convex spherical refracting surfaces.

(vi) The first refracting surface forms image I_1 of the object O [Fig. (b)].

(vii) Image I_1 acts as virtual object for the second surface that forms the image at I [Fig. (c)].

Applying the equation for spherical refracting surface to the first interface ABC, we get

$$\frac{n_1}{OB} + \frac{n_2}{BI_1} = \frac{n_2 - n_1}{BC_1} \quad \ldots\text{(i)}$$

A similar procedure applied to the second interface ADC, we get

$$-\frac{n_2}{DI_1} + \frac{n_1}{DI} = \frac{n_2 - n_1}{DC_2} \quad \ldots\text{(ii)}$$

For a thin lens, $BI_1 = DI_1$

Adding Eqs. (i) and (ii), we get

$$\frac{n_1}{OB} + \frac{n_1}{DI} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \quad \ldots\text{(iii)}$$

Suppose the object is at infinity, i.e.

$$OB \to \infty \quad \text{and} \quad DI \to f$$

So, Eq. (iii) can be written as,

$$\frac{n_1}{f} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \quad \ldots\text{(iv)}$$

The point where image of an object placed at infinity is formed is called the focus (f) of the lens and the distance f gives its focal length. A lens has two foci, F and F' on either side of it by sign convention.

$$BC_1 = R_1$$

or

$$DC_2 = -R_2$$

Therefore, Eq. (iv) can be written as

$$\boxed{\frac{1}{f} = ({}_1n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \quad \left[\because {}_1n_2 = \frac{n_2}{n_1}\right] \ldots\text{(v)}$$

Eq. (v) is known as the **lens Maker's formula**.

Putting $\frac{n_2}{n_1} = n$, refractive index of material of lens w.r.t. its surroundings, we get

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

From Eqs. (iii) and (iv), we get

$$\frac{n_1}{OB} + \frac{n_1}{DI} = \frac{n_1}{f} \quad \ldots\text{(vi)}$$

As, B and D both are close to the optical centre of the lens, $OB = -u$, $DI = +v$, we get

$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}} \quad \ldots\text{(vii)}$$

Eq. (vii) is the **thin lens formula**.

From lens maker's formula, it is clear that focal length of lens depends upon radii of curvature of lens and refractive index of material of lens w.r.t. its surroundings.

EXAMPLE |3| The radii of curvature of the surfaces of a double convex lens are 20 cm and 40 cm, respectively and its focal length is 20 cm. What is refractive index of the material of the lens?

Sol. Given, $R_1 = 20\text{ cm}$, $R_2 = -40\text{ cm}$, $f = 20\text{cm}$, $n = ?$

We know that, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{20} = (n-1)\left(\frac{1}{20} + \frac{1}{40}\right)$$

$$\Rightarrow \frac{1}{n-1} = 20\left(\frac{2+1}{40}\right) = \frac{3}{2}$$

$$\Rightarrow n - 1 = \frac{2}{3} \Rightarrow 3n = 5 \Rightarrow n = \frac{5}{3}$$

Hence, the refractive index of the material of the lens is 5/3.

Dependence of Focal Length on Refractive Index

Refractive index of material of lens depends upon the medium in which it is kept. Generally, the lens is placed in air, so in the above formula, n is the refractive index of material of lens with respect to air. If lens is placed in a medium other than air, then due to change in refractive index (n), focal length of the lens changes. If lens is immersed in a liquid whose refractive index with respect to air is less than the refractive index of material of lens with respect to air, then focal length of the lens increases.

If lens is immersed in a liquid whose refractive index with respect to air is more than the refractive index of material of the lens with respect to air, then focal length will become negative. That means, the nature of lens will change in such a medium, convex lens will behave like concave lens and concave lens will behave like convex lens.

If lens is immersed in a liquid whose refractive index with respect to air is equal to the refractive index of material of lens with respect to air, then focal length of the lens will become infinite and it will behave like plane glass sheet. Also, in such medium, lens will become invisible.

Dependence of Focal Length on the Radii of Curvature

From lens maker's formula, it is clear that the focal length of a lens of large radii of curvature is large and that of a lens of small radii of curvature is small. In simple words, the focal length of thin lens is large and that of thick lens is small. For plano-convex or plano-concave lens, $R_1 = R$ and $R_2 = \infty$ (for plane surface).

Linear Magnification Produced by a Lens (m)

Linear magnification of a lens is defined as, the ratio of the height of the image formed by the lens to height of the object.

$$\text{Linear magnification } (m) = \frac{\text{Height of image } (I)}{\text{Height of object } (O)}$$

For Convex Lens

When image is real, $m = \dfrac{-I}{O} = \dfrac{v}{-u}$

When image is real, it is inverted and forms on the other side of object.

When image is virtual, $m = \dfrac{I}{O} = \dfrac{v}{u}$

When image is virtual, it is erect and forms on the same side of object. Thus, it can be said that convex lens gives positive linear magnification for virtual image and negative linear magnification for real image.

For Concave Lens

Concave lens always forms virtual image, so linear magnification of concave lens, $m = \dfrac{I}{O} = \dfrac{v}{u}$.

Concave lens always gives positive linear magnification. Other formulae for linear magnification are

$$m = \frac{v}{u} = \frac{f - v}{f} = \frac{f}{f + u}$$

EXAMPLE |4| The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. Find the ratio of $\dfrac{m_{25}}{m_{50}}$.

Sol. Since, magnification, $m = \dfrac{f}{f + u}$

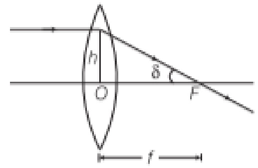
$\Rightarrow \quad m_{25} = \dfrac{20}{20 - 25} = -4$

Similarly, $\quad m_{50} = \dfrac{20}{20 - 50} = \dfrac{-2}{3}$

Therefore, $\quad \dfrac{m_{25}}{m_{50}} = (-4)\left(\dfrac{-3}{2}\right) = 6$

Power of a Lens

The ability of a lens to converge or diverge the rays of light incident on it is called the **power of the lens.**



Power of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from the optical centre.

According to the figure.

$$\tan \delta = \frac{h}{f}, \quad \text{if } h = 1, \text{ then}$$

$$\tan \delta = \frac{1}{f}$$

For small values of δ, $\tan \delta \approx \delta$

$$\therefore \quad \delta = \frac{1}{f}$$

Thus, power of a lens, $P = \dfrac{1}{f}$.

The SI unit of power of lens is dioptre (D). The power of a lens is measured as the reciprocal of its focal length (in metre).

$$P = \frac{1}{f \text{ (in m)}}$$

If $f = 1\,\text{m}$, then $P = 1\,\text{m}^{-1} = 1\,\text{dioptre (D)}$

According to the lens Maker's formula for a lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \left[\because P = \frac{1}{f}\right]$$

We have, $\boxed{P = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}$

Here, R_1 and R_2 are to be measured in metre.

For converging (convex) lens, power is positive and for diverging (concave) lens, power is negative.

EXAMPLE |5| If the radii of curvature of the faces of a double convex lens are 9 cm and 15 cm, respectively and the refractive index of glass is 1.5, then determine the focal length and the power of the lens.

Sol. Given, radii of curvature, $R_1 = 9\text{ cm}$, $R_2 = -15\text{ cm}$, refractive index, $\mu = 1.5$, $f = ?$, $P = ?$

According to lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (1.5 - 1)\left[\frac{1}{9} - \left(\frac{-1}{15}\right)\right]$$

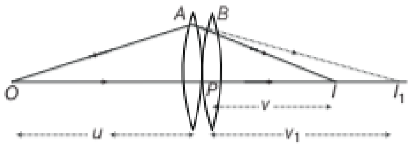
$$\Rightarrow \quad \frac{1}{f} = (0.5)\left(\frac{1}{9}+\frac{1}{15}\right) = (0.5)\left(\frac{5+3}{45}\right)$$

$$\Rightarrow \quad \frac{1}{f} = 0.5 \times \frac{8}{45} \Rightarrow f = \frac{45}{4} = 11.25 \text{ cm}$$

$$\therefore \text{Power,} \quad P = \frac{1}{f} = \frac{1}{11.25 \times 10^{-2}} = \frac{10000}{1125} = 8.88 \text{ D}$$

Combination of Thin Lenses in Contact

Consider two lenses A and B of focal lengths f_1 and f_2 placed in contact with each other. An object is placed at a point O beyond the focus of the first lens A. The first lens produces an image at I_1 (virtual image), which serves as a virtual object for the second lens B, producing the final image at I.



Since, the lenses are thin, we assume the optical centres (P) of the lenses to be co-incident.

For the image formed by the first lens A, we obtain

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \quad \ldots\text{(i)}$$

For the image formed by the second lens B, we get

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \ldots\text{(iii)}$$

If the two lens system is regarded as equivalent to a single lens of focal length f. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}} \quad \ldots\text{(v)}$$

For several thin lenses of focal lengths $f_1, f_2, f_3, \ldots$, the effective focal length is

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots} \quad \ldots\text{(vi)}$$

In terms of power, Eq. (vi) can be written as

$$\boxed{P = P_1 + P_2 + P_3 + \ldots}$$

EXAMPLE |6| Two thin lenses are in contact and the focal length of the combination is 80 cm. If the focal length of one lens is 20 cm, then what would be the power of the another lens?

Sol. Given, combined focal length, $F = 80$ cm,

$f_1 = 20$ cm, $P_2 = ?$

$$\therefore \quad P = \frac{100}{F \text{ (cm)}} = \frac{100}{80} = 1.25 \text{ D}$$

$$\Rightarrow \quad P_1 = \frac{100}{f_1} = \frac{100}{20} = 5 \text{ D}$$

We know that, $P_1 + P_2 = P$

$$\therefore \quad P_2 = P - P_1 = 1.25 - 5 = -3.75 \text{ D}$$

Magnification by Combination of Lenses

Suitable combination of lenses helps to obtain diverging or converging lens of desired magnification. It also enhances sharpness of the image. Since, the image formed by the first lens becomes the object for second lens and so on. So, the magnification of combination (m) is the product of magnification (m_1, m_2, m_3) of individual lenses.

Magnification of combination of lenses,

$$m = m_1 \times m_2 \times m_3 \times \ldots$$

(i) If combination of lenses consists of one convex lens (f_1) and one concave lens ($-f_2$), then

for combination of lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{-f_2} = \frac{1}{f_1} - \frac{1}{f_2}$$

$$\Rightarrow \quad \boxed{f = \frac{f_1 f_2}{f_2 - f_1}}$$

(ii) If $f_1 > f_2$, then f is negative, i.e. combination will behave like concave lens, when focal length of convex lens is larger. If $f_1 < f_2$, then f is positive, i.e. combination will behave like convex lens, when focal length of convex lens is smaller. If $f_1 = f_2$, then f is infinite, i.e. combination will behave like plane glass sheet.

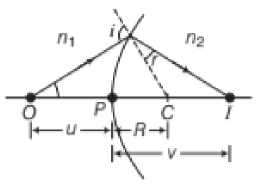
If the lenses are placed d distance apart, then

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. For the refraction shown below the correct relation is,



(a) $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ (b) $\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_2 - n_1}{R}$

(c) $\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}$ (d) $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_1 - n_2}{R}$

2. Light from a point source in air falls on a spherical glass surface ($n = 1.5$ and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. Image distance from the glass surface is

(a) 20 cm (b) 50 cm

(c) 100 cm (d) 75 cm

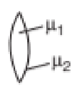
3. First and second focal lengths of spherical surface of n refractive index are f_1 and f_2 respectively. The relation between them, is

(a) $f_2 = f_1$ (b) $f_2 = -f_1$ (c) $f_2 = nf_1$ (d) $f_2 = -nf_1$

4. A magician during a show makes a glass lens with $n = 1.47$ disappear in a trough of liquid. Refractive index of the liquid is

(a) 1.47 (b) 1.33 (c) $\frac{4}{3}$ (d) $\frac{12}{5}$

5. Which of the following is true for rays coming from infinity?



(a) Two images are formed

(b) Continuous image is formed between focal points of upper and lower lens

(c) One image is formed

(d) None of the above

6. Two thin lenses are in contact and that combination has 15 cm focal length. If one lens has focal length 30 cm, then what is the second lens focal length?

(a) 15 cm (b) 25 cm

(c) 20 cm (d) 30 cm

7. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will **NCERT Exemplar**

(a) act as a convex lens only for the objects that lie on its curved side

(b) act as a concave lens for the objects that lie on its curved side

(c) act as a convex lens irrespective of the side on which the object lies

(d) act as a concave lens irrespective of side on which the object lies

8. Two lenses are in contact having focal length 25 cm and – 40 cm. Find power of this combination.

(a) – 6.67 D (b) – 2.5 D (c) + 1.5 D (d) + 4 D

9. Two lenses are in contact having powers of 5 D and – 3 D. The focal length of this combination will be

(a) 50 cm (b) 75 cm (c) 25 cm (d) + 20 cm

VERY SHORT ANSWER Type Questions

10. A beam of light is converging towards a certain point. A parallel sided glass plate is introduced in the path of the converging beam. How will the point of convergence be shifted?

11. What type of lens is an air bubble inside water?

12. A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens? **All India 2015**

13. A biconvex lens made of a transparent material of refractive index 1.25 is immersed in water of refractive index 1.33. Will the lens behave as a converging or a diverging lens? Give reason. **All India 2014**

14. Under what condition, does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid? **Delhi 2012**

15. A glass lens is immersed in water. How is power of the lens affected?

SHORT ANSWER Type Questions

16. The lens shown in the given figure is made of two different materials. A point object is placed on the principal axis of this lens. How many images will be obtained?

17. Show analytically from the lens equation that when the object is at the principal focus, the image is formed at infinity.

18. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. What will be the graph drawn between u and v?

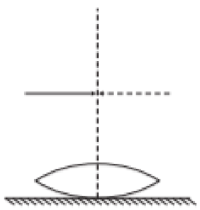
19. A magician during a show makes a glass lens $n = 1.47$ disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

LONG ANSWER Type I Questions

20. An equiconvex lens of focal length f is cut into two equal halves in thickness. What is the focal length of each half?

21. Define power of a lens. Write its units. Deduce the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ for two thin lenses kept in contact coaxially. **Foreign 2012**

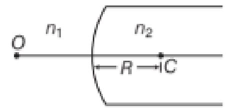
22. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5, is placed on a layer of liquid placed on the top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x. On removing the liquid layer and repeating the experiment, the distance is found to be y. Obtain the expression for the refractive index of the liquid in terms of x and y. **CBSE 2018**



23. The objective of an astronomical telescope has a diameter of 150 mm and a focal length of 4 m. The eyepiece has a focal length of 25 mm. Calculate the magnifying and resolving power of telescope ($\lambda = 6000$ Å for yellow colour). **Delhi 2011**

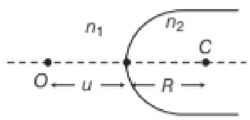
LONG ANSWER Type II Questions

24. Figure shows a convex spherical surface with centre of curvature C, separating the two media of refractive indices n_1 and n_2. Draw a ray diagram showing the formation of the image of a point object O lying on the principal axis. Derive the relationship between the object and image distance in terms of refractive indices of the media and the radius of curvature R on the surface.



All India 2014

25. (i) A point object O is kept in a medium of refractive index n_1 in front of a convex spherical surface of radius of curvature R which separates the second medium of refractive index n_2 from the first one, as shown in the figure.
Draw the ray diagram showing the image formation and deduce the relationship between the object distance and the image distance in terms of n_1, n_2 and R.



(ii) When the image formed above acts as a virtual object for a concave spherical surface separating the medium n_2 from $n_1 (n_2 > n_1)$, draw this ray diagram and write the similar [similar to (i)] relation. Hence obtain the expression for the lens Maker's formula. **All India 2015**

NUMERICAL PROBLEMS

26. A converging lens of refractive index 1.5 is kept in a liquid medium having the same refractive index. What would be the focal length of lens in the medium?

27. The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. If focal length of the lens is 12 cm, find the refractive index of the material of the lens. **Delhi 2010**

28. Find the radius of curvature of the convex surface of a plano-convex lens, whose focal length is 0.3 m and the refractive index of the material of the lens is 1.5. **Delhi 2010**

29. A biconvex lens has a focal length 2/3 times the radius of curvature of either surface. Calculate the refractive index of lens material. **Delhi 2010**

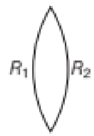
30. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a

converging or a diverging lens? Ignore thickness of the lenses. **NCERT**

31. (i) Monochromatic light of wavelength 589 nm is incident from air on a water surface. If μ for water is 1.33, find the wavelength, frequency and speed of the refracted light.

(ii) A double convex lens is made of a glass of refractive index 1.55 with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm. **All India 2017**

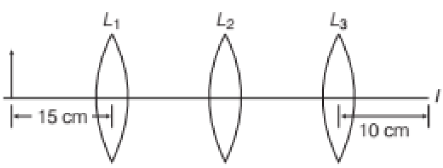
32. Double convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required, if the focal length is to be 20 cm?



NCERT

33. The image obtained with a convex lens is erect and its length is four times the length of the object. If the focal length of the lens is 20 cm, calculate the object and image distances. **All India 2010**

34. You are given three lenses L_1, L_2 and L_3 each of focal length 10 cm. An object is kept at 15 cm in front of L_1, as shown in figure. The final real image is formed at the focus of L_3. Find the separation between L_1, L_2 and L_3. **All India 2012**



HINTS AND SOLUTIONS

1. (a) As refraction formula for curved surface is

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

2. (c) Here, $u = -100$ cm, $v = ?$, $R = +20$ cm, $n_1 = 1$ and $n_2 = 1.5$

As, refraction formula for curved surface, we have

$$\frac{1.5}{v} + \frac{1}{100} = \frac{0.5}{20} \Rightarrow v = +100 \text{ cm}$$

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

3. (b) When medium is equal on both sides of lens, then the numerical value of both focal length is equal, hence $f_2 = -f_1$.

4. (a) The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This give $1/f = 0$ or $f \to \infty$.

5. (a) Since, lens is made of two layers of different refractive indices, for a given wavelength of light it will have two different focal lengths or will have two images at two different points as $\frac{1}{f} \propto (\mu - 1)$ (from Lens maker's formula).

6. (d) Given, $F = 15$ cm, $f_1 = 30$ cm

We know that, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\Rightarrow \quad \frac{1}{15} = \frac{1}{30} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{15} - \frac{1}{30} = \frac{2-1}{30}$$

$$\Rightarrow \quad f_2 = 30 \text{ cm}$$

7. (c) Here, $R = 20$cm, $\mu = 1.5$, on substituting the values in

$f = \frac{R}{\mu - 1} = \frac{20}{1.5 - 1} = 40$ cm of converging nature as f>0.

Therefore, lens act as a convex lens irrespective of the side on which the object lies.

8. (c) Given, $f_1 = 25$ cm, $f_2 = -40$ cm

$$\therefore \quad P_1 = \frac{100}{f_1} = \frac{100}{25} = +4 \text{ D}$$

and $P_2 = \frac{100}{f_2} = \frac{100}{-40} = -2.5 \text{ D}$

$$\therefore \quad P = P_1 + P_2 = 4 + (-2.5) \text{ D} = +1.5 \text{ D}$$

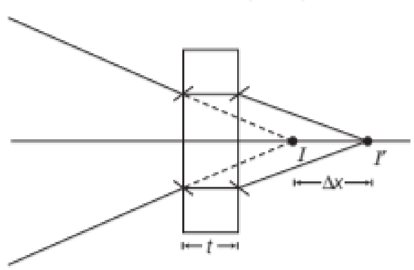
9. (a) Given, $P_1 = 5$ D, $P_2 = -3$ D

$$\therefore \quad P = P_1 + P_2 = 5 + (-3) = 2 \text{ D}$$

$$\therefore \quad P = \frac{1}{f} \Rightarrow 2 = \frac{1}{f}$$

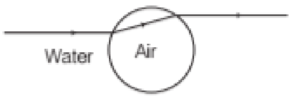
$$\Rightarrow \quad f = \frac{1}{2} \text{m} = \frac{100}{2} \text{cm} = 50 \text{ cm}$$

10. Here, shift is given as $\Delta x = \left(1 - \frac{1}{\mu}\right)t$



which takes place in the direction of ray.

11. It is clearly visible that air bubble acts as a diverging lens (concave lens) in water.



12. A concave lens behaves as a diverging lens, when it is placed in a medium of refractive index less than the refractive index of the material of the lens and behaves as a converging lens, when it is placed in a medium of refractive index greater than the refractive index of the material of the lens.

In the given case, concave lens is immersed in a medium having refractive index greater than the refractive index of the material of the lens $(1.65 > 1.5)$. Therefore, it will behave as a converging lens.

13. When a lens is placed in a liquid, where refractive index is more than that of the material of lens, then the nature of the lens changes. So, when a biconvex lens of refractive index 1.25 is immersed in water (refractive index 1.33), i.e. in the liquid of higher refractive index, its nature will change. So, biconvex lens will act as converging or diverging lens.

14. When refractive index of lens is equal to the refractive index of liquid, it will behave like plane glass sheet.

15. We know that, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For glass $n_2 = 1.5$, for air, $n_1 = 1$, for water $n = 1.33$

$$\therefore \quad \frac{1}{f} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

So, focal length becomes 4 times, hence power becomes $\frac{1}{4}$th of the initial value.

16. Since, refractive index of each material is different, so the lens will have two different focal lengths, one for each material. Hence, two images will formed.

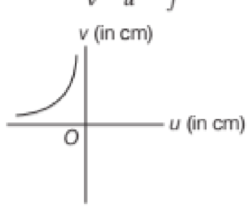
17. Given, $u = -f$

$\therefore$ Lens equation is, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{f} = \frac{1}{f} \Rightarrow \frac{1}{v} = 0$$

$$\Rightarrow \quad v = \frac{1}{0} = \text{infinity}$$

18. As we know that, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



$\therefore$ Correct answer is 1.

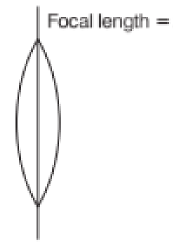
19. If $\mu_1 = \mu_2$, then $f = \infty$

Hence, the lens in the liquid acts like a plane sheet, when refractive index of the lens and the surrounding medium is the same. Therefore, $\mu_1 = \mu_2 = 1.47$.

Hence, the liquid medium is not water, refractive index for water = 1.33.

20. Focal length can be given as

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



where, μ is the refractive index of the lens medium. R_1 and R_2 are radii of curvature.

Equiconvex lens have the same radius of curvature,

i.e. $\quad R_1 = -R_2$

$$\therefore \quad \frac{1}{f'} = (\mu - 1)\left[\frac{1}{R} - \left(-\frac{1}{R}\right)\right] \Rightarrow \frac{1}{f'} = \frac{2(\mu - 1)}{R}$$

$$\therefore \quad f' = 2f$$

Hence, focal length of each half becomes twice of the original value.

21. Refer to text on pages 374 and 375.

22. First measurement gives the focal length $(f_{eq} = x)$ combination of the convex lens and the plano-convex liquid lens. Second measurement gives the focal length $(f_1 = y)$ of the convex lens.

Focal length (f_2) of plano-convex lens is given by

$$\frac{1}{f_2} = \frac{1}{f_{eq}} - \frac{1}{f_1} = \frac{1}{x} - \frac{1}{y}$$

$$\Rightarrow \quad f_2 = \frac{xy}{y - x} \quad \text{...(i)}$$

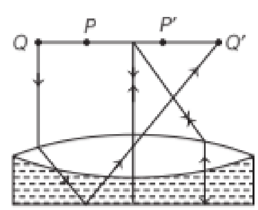
For equiconvex glass lens using Lens Maker's formula, we get

$$\frac{1}{f_1} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{y} = (1.5 - 1)\left(\frac{2}{R}\right)$$

(As $R_1 = R$ and $R_2 = -R$)

$$\Rightarrow \quad \frac{1}{y} = \frac{1}{2} \times \frac{2}{R} \Rightarrow R = y$$



Now, we apply Lens Maker's formula for plano-convex lens.

Here $R_1 = R$ and $R_2 = \infty$ and let n_l = refractive index of liquid

$$\frac{1}{f_2} = (n_l - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = (n_l - 1)\left(\frac{1}{R}\right)$$

$$\Rightarrow \quad n_l = 1 + \frac{R}{f_2} = 1 + \frac{y}{\left(\frac{xy}{y-x}\right)}$$

$$= 1 + \frac{y-x}{x} = \frac{y}{x}$$

23. The diameter of objective of the telescope

$$= 150 \times 10^{-3} \text{ m}, f_o = 4 \text{ m}$$

$$f_e = 25 \times 10^{-3} \text{ m and } D = 0.25 \text{ m}$$

Magnifying power, $m = -\frac{f_o}{f_e}\left(1 + \frac{D}{f_e}\right)$

$$= -\frac{4}{25 \times 10^{-3}}\left(1 + \frac{0.25}{25 \times 10^{-3}}\right) = -1760$$

Now, $d\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 6 \times 10^{-7}}{0.25}$

$$= 2.9 \times 10^{-6} \text{ rad}$$

$\therefore$ Resolving power $= \frac{1}{d\theta} = \frac{1}{2.9 \times 10^{-6}}$

$$= 0.34 \times 10^{6}$$

24. Refer to text on pages 369 and 370.

25. Let a spherical surface separate a rarer medium of refractive index n_1 from the second medium of refractive index n_2. Let C be the centre of curvature and $R = MC$ be the radius of the surface.

Consider a point object O lying on the principal axis of the surface. Let a ray starting from O incident normally on the surface along OM and pass straight. Let another ray of light incident on NM along ON and refract along NI. From M, draw MN perpendicular to OI.

The above figure shows the geometry of the formation of image I of an object O and the principal axis of a spherical surface with centre of curvature C and radius of curvature R.

Here, we have to make following assumptions,

(i) the aperture of the surface is small as compared to the other distance involved.

(ii) NM will be taken as nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\tan \angle NOM = \frac{MN}{OM}, \quad \tan \angle NCM = \frac{MN}{MC}$$

$$\tan \angle NIM = \frac{MN}{MI}$$

For ΔNOC, is the exterior angle.

$$\therefore \quad \angle i = \angle NOM + \angle NCM$$

For small angles, $i = \frac{MN}{OM} + \frac{MN}{NC}$ ...(i)

Similarly, $r = \angle NCM - \angle NIM$

$$\Rightarrow \quad r = \frac{MN}{NC} - \frac{MN}{NI} \quad \text{...(ii)}$$

By Snell's law, we get

$$n_1 \sin i = n_2 \sin r$$

For small angles, $n_1 i = n_2 r$

Put the values of i and r from Eqs. (i) and (ii), we get

$$n_1\left(\frac{MN}{OM} + \frac{MN}{MC}\right) = n_2\left(\frac{MN}{MC} - \frac{MN}{MI}\right)$$

$$\Rightarrow \quad \frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \quad \text{...(iii)}$$

Applying new cartesian sign conventions, we get

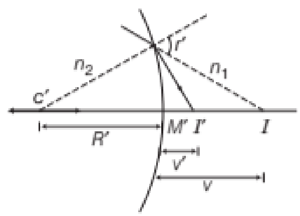
$$OM = -u, \; MI = +v$$

and $MC = +R$

Substituting this in Eq. (iii), we get

$$\frac{n_2}{v} - \frac{n_2}{u} = \frac{n_2 - n_1}{R} \quad \text{...(iv)}$$

Now, the image I' acts as a virtual object for the second surface that will form a real at I. As, refraction takes place from denser to rarer medium,



$$\therefore \quad \frac{-n_2}{v} + \frac{n_1}{v'} = \frac{n_2 - n_1}{-R^1} \quad \text{...(v)}$$

On adding Eqs. (iv) and (v), we get

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R} - \frac{1}{R'}\right) \quad \left[\because n_{21} = \frac{n_2}{n_1}, \frac{1}{f} = \frac{1}{v} - \frac{1}{u}\right]$$

26. When lens is immersed in a liquid, then

$$\frac{1}{f_L} = ({}^{L}\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, ${}^{L}\mu_g$ = refractive index of lens material (glass) w.r.t. liquid.

$$\therefore \quad \frac{\mu_g}{\mu_L} = \frac{1.5}{1.5} = 1$$

Hence, $\frac{1}{f_L} = (1-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0 \Rightarrow f_L = \infty$

27. Given, $R_1 = +10$ cm, $R_2 = -15$ cm, $f = +12$ cm, $\mu = ?$

Applying lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{12} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{15}\right) = (\mu - 1)\frac{5}{30}$$

$$\Rightarrow \quad (\mu - 1) = \frac{1}{2} \Rightarrow \mu = \frac{3}{2}$$

28. For a plano-convex lens, $R_1 = \infty$

$R_2 = -R$, $f = 0.3$ m $= 30$ cm

$\mu = 1.5$

Radius of curvature of plano-convex lens, $R = ?$

Applying lens Maker's formula, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{30} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$= \frac{(1.5 - 1)}{R} \Rightarrow R = 15 \text{ cm}$$

29. Given, $f = \frac{2}{3}R$, $R_1 = +R$, $R_2 = -R$

$\therefore$ Using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{3}{2R} = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\Rightarrow \quad \mu - 1 = \frac{3}{4}$$

$$\Rightarrow \quad \mu = 1 + \frac{3}{4} = \frac{7}{4}$$

30. Given, focal length of convex lens, $f_1 = 30$ cm

Focal length of concave lens, $f_2 = -20$ cm

Using the formula of combination of lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

$$\Rightarrow \quad f = -60 \text{ cm}$$

Since, the focal length of combination is negative in nature. So, the combination behaves like a diverging lens, i.e. as a concave lens.

31. (i) In refraction, frequency remains same, so

$f_{\text{refracted beam}} = f_{\text{incident beam}}$

Also, $\mu_{21} = \frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \frac{\lambda_1}{\lambda_2}$ $[\because v = f\lambda]$

$$\Rightarrow v_2 = \frac{v_1}{\mu_{21}} = \frac{3 \times 10^8}{1.33} = 2.25 \times 10^8 \text{ ms}^{-1}$$

$$\therefore \quad \lambda_2 = \frac{\lambda_1}{\mu_{21}} = \frac{589}{1.33} = 442.85 \approx 443 \text{ nm}$$

So, wavelength of reflected beam ≈ 443 nm and its speed $= 2.25 \times 10^8$ ms^{-1}

(ii) For a biconvex lens, using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $f = 20$ cm, $\mu = 1.55$

$\Rightarrow \quad R_1 = +R$ and $R_2 = -R$

We have, $\frac{1}{f} = (\mu - 1)\frac{2}{R}$

$$\Rightarrow \quad R = 2(\mu - 1)f = 2 \times (1.55 - 1) \times 20 = 22 \text{ cm}$$

$\therefore$ Radius of 22 cm is required.

32. Given, the refractive index of glass with respect to air,

$${}^a\mu_g = 1.55$$

For double convex lenses, $R_1 = R$, $R_2 = -R$

[$\because$ both faces have same radius of curvature]

[for double convex lens, one radius is taken as positive and other negative]

Focal length of lens, $f = +20$ cm

Using the lens Maker's formula,

$$\frac{1}{f} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{20} = (1.55 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{20} = 0.55 \times \frac{2}{R} \Rightarrow R = 0.55 \times 2 \times 20 = 22 \text{ cm}$$

Thus, the required radius of curvature is 22 cm.

33. As magnification, $m = \frac{I}{O} = \frac{v}{u} \Rightarrow I = 4 \times$ length of object

$$\Rightarrow \quad \frac{I}{O} = 4 \Rightarrow \frac{v}{u} = 4 \Rightarrow v = 4u$$

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{(-4u)} - \frac{1}{(-u)}$

$$\Rightarrow \quad \frac{1}{f} = -\frac{1}{4u} + \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{4-1}{4u} = \frac{3}{4u}$$

$$\Rightarrow \quad u = \frac{20 \times 3}{4} = 15 \text{ cm}$$

$$\Rightarrow \quad v = 4u = 15 \times 4 = 60 \text{ cm}$$

Distance of the object, $u = 15$ cm

Distance of the image, $v = 60$ cm

The image is on the same side of the object.

34. For lens L_1, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Given, $u = -15$ cm, $f = +10$ cm, $v = ?$

$$\therefore \quad \frac{1}{10} = \frac{1}{v} + \frac{1}{15} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{15} \Rightarrow \frac{1}{v} = \frac{1}{30}$$

Distance of image from lens L_1, $v = 30$ cm

For lens L_3, $\frac{1}{f''} = \frac{1}{v''} - \frac{1}{u''}$

Distance of image from lens L_3, $v'' = 10$ cm

$$\therefore \quad \frac{1}{10} = \frac{1}{10} - \frac{1}{u''} \Rightarrow \frac{1}{u''} = 0 \Rightarrow u'' = \infty$$

The refracted rays from lens L_2 becomes parallel to principal axis. It is possible only when image formed by L_1 lies at first focus of L_2, i.e. at a distance of 10 cm from L_2.

$\therefore$ Separation between L_1 and $L_2 = 30 + 10 = 40$ cm

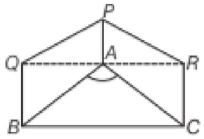
The distance between L_2 and L_3 may take any value.

|TOPIC 4|

Prism and Optical Instruments

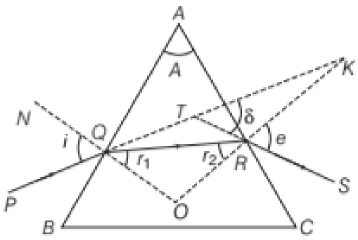
A **prism** is a portion of a transparent medium bounded by two plane faces inclined to each other at a suitable angle.

In the given figure, $ABQP$ and $ACRP$ are the two refracting faces and $\angle A$ is called angle of prism.



REFRACTION OF LIGHT THROUGH A PRISM

The figure below shows the passage of light through a triangular prism ABC.



The angles of incidence and refraction at first face AB are i and r_1.

The angle of incidence at the second face AC is r_2 and the angle of emergence is e.

The angle between the emergent ray RS and incident ray PQ is called **angle of deviation** (δ).

Here, $\angle PQN = i, \angle SRK = e$

$\angle RQO = r_1, \angle QRO = r_2$

$\angle KTS = \delta, \angle TQO = i$

and $\angle TQR = i - r_1$

or $\angle TRQ = e - r_2$

In ΔTQR, the side QT has been produced outwards. Therefore, the exterior angle δ should be equal to the sum of the interior opposite angles.

i.e. $\delta = \angle TQR + \angle TRQ$

$= (i - r_1) + (e - r_2)$

$\Rightarrow \quad \delta = (i + e) - (r_1 + r_2) \quad \ldots(i)$

In ΔQRO,

$r_1 + r_2 + \angle ROQ = 180° \quad \ldots(ii)$

From quadrilateral $AROQ$, we have the sum of angles

$\angle AQO + \angle ARO = 180°$.

This means that the sum of the remaining two angles should be 180°.

i.e. $\angle A + \angle ROQ = 180° \quad \ldots(iii)$

[$\angle A$ is called the angle of prism]

From Eqs. (ii) and (iii), we get

$r_1 + r_2 = A \quad \ldots(iv)$

Substituting the value from Eq. (iv) in Eq. (i), we obtain

$\delta = (i + e) - A$

If μ is the refractive index of material of the prism, then according to Snell's law,

$$\mu = \frac{\sin i_1}{\sin r_1}$$

When angles are small, $\sin i_1 \approx i_1$ and $\sin r_1 \approx r_1$

$\therefore \quad \mu = \frac{i_1}{r_1} \Rightarrow i = \mu r_1 \quad$ [here, $i_1 = i$]

Similarly, $\quad \mu = \frac{i_2}{r_2}$ or $\mu = \frac{e}{r_2} \quad [\because i_2 = e]$

$\Rightarrow \quad e = \mu r_2$

$\therefore \quad \delta = i + e - A$

$= \mu r_1 + \mu r_2 - A = \mu(r_1 + r_2) - A$

But $\quad r_1 + r_2 = A$

$\therefore \quad \delta = \mu A - A$

or $\quad \boxed{\delta = (\mu - 1) A}$

This is the angle through which a ray deviates on passing through a thin prism of small refracting angle A.

EXAMPLE |1| A thin prism of 5° angle gives a deviation of 3.2°. What is the value of refractive index of the material of the prism?

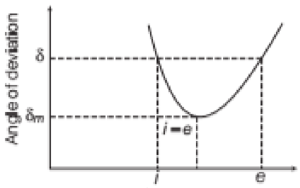
Sol. Given, $A = 5°, \delta = 3.2°, \mu = ?$

We know that, $\delta = A(\mu - 1)$

$\therefore \quad \mu = 1 + \frac{\delta}{A} = 1 + \frac{3.2°}{5°} = 1 + 0.64 = 1.64$

Prism Formula

If the angle of incidence is increased gradually, then the angle of deviation first decreases, attains a minimum value (δ_m) and then again starts increasing.



When angle of deviation is minimum, the prism is said to be placed in the minimum deviation position. There is only one angle of incidence for which the angle of deviation is minimum.

When $\delta = \delta_m$ [prism in minimum deviation position]

$$e = i \quad \text{and} \quad r_2 = r_1 \qquad \text{...(i)}$$

$\because \quad r_1 + r_2 = A$

$\Rightarrow \quad r + r = A \text{ or } r = \dfrac{A}{2}$

Also, we have

$$A + \delta = i + e \qquad \text{...(ii)}$$

Putting $\delta = \delta_m$ and $e = i$ in Eq. (ii), we get

$$A + \delta_m = i + i$$

$$\Rightarrow \quad i = \left(\frac{A + \delta_m}{2}\right)$$

From Snell's law, $\mu = \dfrac{\sin i}{\sin r}$

$$\therefore \quad \boxed{\mu = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin \dfrac{A}{2}}}$$

This relation is called a **prism formula.**

For thin prisms (i.e. A is very small), the value of δ_m is also very small.

So, $$\mu = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin \dfrac{A}{2}} \approx \frac{\dfrac{A + \delta_m}{2}}{A/2}$$

$$\Rightarrow \quad \delta_m = (\mu - 1)A$$

EXAMPLE |2| A ray of light suffers minimum deviation, while passing through a prism of refractive index 1.5 and refracting angle 60°. Calculate the angle of deviation and angle of incidence.
(Given, $\sin^{-1}(0.75) = 48.6°$)

Sol. Given, refractive index, $\mu = 1.5$

Angle of prism, $A = 60°$, angle of deviation, $\delta_m = ?$

Angle of incidence, $i = ?$

We know that, $\mu = \dfrac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$$\Rightarrow \quad 1.5 = \frac{\sin\left(\dfrac{60° + \delta_m}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)}$$

$$\Rightarrow \quad 1.5 \sin 30° = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow \quad 1.5 \times 0.5 = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow \quad \frac{60° + \delta_m}{2} = \sin^{-1}(0.75)$$

$$\Rightarrow \quad \frac{60° + \delta_m}{2} = 48.6°$$

$$\delta_m = 48.6° \times 2 - 60° = 37.2°$$

Also, the angle of incidence,

$$i = \frac{(A + \delta_m)}{2} = \frac{60° + 37.2°}{2} = 48.6°$$

OPTICAL INSTRUMENTS

Using the reflecting and refracting properties of mirrors, lenses and prisms, many optical instruments have been designed like microscopes and telescopes. Our eye is a natural optical device.

The Eye

The structure and working of eye were already learnt in your younger classes. The **eye lens** is a convex lens whose focal length can be modified by the ciliary muscles. This property of eye is called **accommodation.** The image is formed on a film of nerve fibres called **retina.**

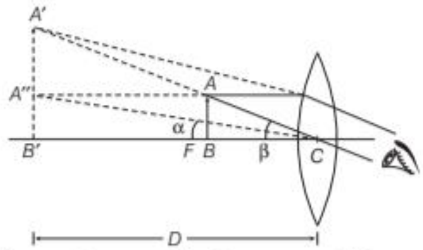
The closest distance for which the lens can form image is called the **near point** and its value is 25 cm for a normal eye. The far point of a normal eye is infinity. It is the farthest point upto which the eye can see clearly.

Simple Microscope

Microscope is an optical instrument which forms large image of close and minute objects. A simple microscope is a converging lens of small focal length. When an object is at a distance less than the focal length of the lens, the image obtained is virtual, erect and magnified.

When the object is at a distance equal to the focal length of the lens, the image is formed at infinity.

Case I **When the image is formed at the near point**



The **angular magnification** or **magnifying power** of a simple microscope is defined as the ratio of the angle β subtended at the eye by image at the near point and the angle α subtended at the unaided eye by the object at the near point.

$\therefore$ Magnifying power, $m = \dfrac{\beta}{\alpha}$ ...(i)

In $\Delta A'B'C$, $\tan\beta = \dfrac{A'B'}{D}$

In $\Delta A''B''C$, $\tan\alpha = \dfrac{A''B'}{D} = \dfrac{AB}{D}$

Since, the angles are small, then

$\tan\alpha \approx \alpha$ and $\tan\beta \approx \beta$

$\therefore \quad \beta = \dfrac{A'B'}{D}$ and $\alpha = \dfrac{AB}{D}$

From Eq. (i), we have

$$m = \frac{A'B'}{D} \times \frac{D}{AB} = \frac{A'B'}{AB}$$

This gives the linear magnification produced by the lens.

It can be proved that, $\dfrac{A'B'}{AB} = \dfrac{v}{u}$

We know that, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Multiplying both sides by v, we have $\dfrac{v}{v} - \dfrac{v}{u} = \dfrac{v}{f}$

$$\Rightarrow \quad 1 - m = \frac{v}{f} \Rightarrow m = 1 - \frac{v}{f}$$

$$\therefore \quad \boxed{m = 1 + \frac{D}{f}}$$

[$\because v = -D$ because image is formed at near point]

In this case, the eye is placed behind the lens at a distance a, then

$$\boxed{m = 1 + \frac{D - a}{f}}$$

Case II **When the image is formed at infinity**

i.e. $v = \infty$

In this case, $\beta = \dfrac{AB}{f}$ and $\alpha = \dfrac{AB}{D}$

$$\therefore \quad m = \frac{AB}{f} \times \frac{D}{AB} = \frac{D}{f}$$

$$\boxed{m = \frac{D}{f}}$$

EXAMPLE |3| A convex lens of focal length 5 cm is used as a simple microscope. What will be the magnifying power when the image is formed at the least distance of distinct vision?

Sol. Given, focal length, $f = 5$ cm

Least distance of distinct vision, $D = 25$ cm

$\therefore$ Magnification, $m = \left(1 + \dfrac{D}{f}\right) = \left(1 + \dfrac{25}{5}\right) = 6$

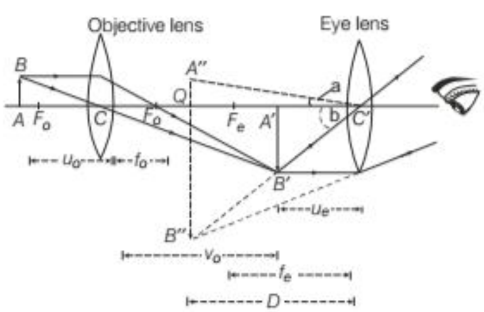
Compound Microscope

A compound microscope consists of two convex lenses coaxially separated by some distance. The lens nearer to the object is called the **objective**. The lens through which the final image is viewed is called the **eyepiece**.

Working

The objective of compound microscope forms the real, inverted and magnified image of the object. This image serves as the object for the second lens, i.e. eyepiece which produces the final image, which is enlarged and virtual.

The first inverted image is thus near the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity or a little closer for image formation at the near point. The final image is inverted with respect to the original object.



Angular magnification or **magnifying power** of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when both are placed at least distance of distinct vision.

$\therefore$ Angular magnification, $m = \dfrac{\beta}{\alpha}$

Since, the angles are small, then

$$\alpha \approx \tan\alpha \quad \text{or} \quad \beta \approx \tan\beta$$

$$\therefore \quad m = \frac{\tan\beta}{\tan\alpha} \quad \ldots\text{(i)}$$

From right angled $\Delta C'QB''$, we have

$$\tan\beta = \frac{B''Q}{C'Q} = \frac{B''Q}{D} = \frac{A''B''}{D}$$

Also, from right angled $\Delta C'A''Q$, we have

$$\tan\alpha = \frac{A''Q}{C'Q} = \frac{AB}{D} \quad [\because A''Q = AB]$$

Substituting the values of $\tan\alpha$ and $\tan\beta$ in Eq. (i), we have

$$m = \frac{B''Q}{D} \times \frac{D}{AB} = \frac{B''Q}{AB}$$

$$\Rightarrow \quad m = \frac{B''Q}{A'B'} \times \frac{A'B'}{AB}$$

Thus, the magnification produced by the compound microscope is the product of the magnification produced by the eyepiece and objective.

$$\therefore \quad m = m_e \times m_o \quad \ldots\text{(ii)}$$

where, m_e and m_o are the magnifying powers of the eyepiece and objective, respectively.

The linear magnification of the real inverted image produced by the eyepiece is $\dfrac{A''B''}{A'B'}$.

Case I **When the final image is formed at near point**

Linear magnification is given by

$$m_e = 1 + \frac{D}{f_e} \quad \ldots\text{(iii)}$$

where, f_e is focal length of the eyepiece

$\dfrac{A'B'}{AB}$ is the linear magnification of the object produced by the objective.

$$m_o = \frac{v_o}{u_o} \quad \ldots\text{(iv)}$$

From Eqs. (ii), (iii) and (iv), we have

$$m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) \quad \ldots\text{(v)}$$

We know that, $\dfrac{1}{v_o} - \dfrac{1}{u_o} = \dfrac{1}{f_o}$

Multiplying both sides by v_o, we have

$$\frac{v_o}{v_o} - \frac{v_o}{u_o} = \frac{v_o}{f_o}$$

$$\Rightarrow \quad -\frac{v_o}{u_o} = -1 + \frac{v_o}{f_o} \Rightarrow \frac{v_o}{u_o} = 1 - \frac{v_o}{f_o}$$

Substituting the value of $\dfrac{v_o}{u_o}$ in Eq. (v), we have

$$\boxed{m = \left(1 - \frac{v_o}{f_o}\right)\left(1 + \frac{D}{f_e}\right)}$$

Case II **When the final image is at infinity**

If u_o is the distance of the object from the objective and v_o is the distance of the image from the objective, then the magnifying power of the objective is $m_o = \dfrac{v_o}{u_o}$

When the final image is at infinity, then angular magnification is given by

$$\boxed{m_e = \frac{D}{f_e}}$$

The total magnification when image is at infinity is given by

$$m = m_o \times m_e = \left(\frac{v_o}{u_o} \times \frac{D}{f_e}\right)$$

If the object is very close to the principal focus of the objective and the image formed by the objective is very close to the eyepiece, then

$$m = \frac{-L}{f_o} \cdot \frac{D}{f_e}$$

where, L = length of the tube of microscope

In this case, the microscope is said to be in normal adjustment.

EXAMPLE |4| A compound microscope has an objective of focal length 1 cm and an eyepiece of focal length 2.5cm. An object has to be placed at a distance of 1.2 cm away from the objective for the normal adjustment. Determine the angular magnification and length of microscope tube.

Sol. Given, focal length of objective, $f_o = 1$ cm

Focal length of eyepiece, $f_e = 2.5$ cm

Object distance, $u_o = -1.2$ cm

$$\because \quad \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\Rightarrow \quad \frac{1}{v_o} = \frac{1}{u_o} + \frac{1}{f_o}$$

$$\Rightarrow \quad \frac{1}{v_o} = 1 - \frac{1}{1.2} = \frac{0.2}{1.2}$$

$$\Rightarrow \quad v_o = \frac{1.2}{0.2} \Rightarrow v_o = 6\text{ cm}$$

$\because$ Angular magnification, $m = \frac{v_o}{|u_o|}\left(1 + \frac{D}{f_e}\right)$

$$\Rightarrow \quad m = \frac{6}{|-1.2|}\left(1 + \frac{25}{2.5}\right) = 55$$

$\therefore$ Length of microscope tube,

$$L = v_o + f_e = (6 + 2.5) = 8.5\text{ cm}$$

Astronomical (Refracting) Telescope

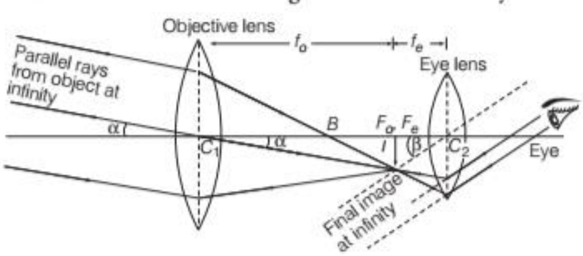
An astronomical telescope is an optical instrument which is used for observing distinct images of heavenly bodies like stars, planets, etc., when the final image is formed at infinity.

Astronomical telescope has two convex lenses coaxially separated by some distance. The lens towards the object is called **objective** and has much larger aperture than the eyepiece of the lens towards the eye.

Working

Light from the distant object enters the objective and real image is formed at second focal point of objective. The eyepiece magnifies this image producing a final inverted image.

***Case* I When the final image is formed at infinity**



Angular magnification is given by

$$m = \frac{\beta}{\alpha}$$

Since, β and α are very small.

$$\therefore \quad \beta \approx \tan\beta$$

$$\text{or} \quad \alpha \approx \tan\alpha$$

$$\Rightarrow \quad m = \frac{\tan\beta}{\tan\alpha} \quad \ldots\text{(i)}$$

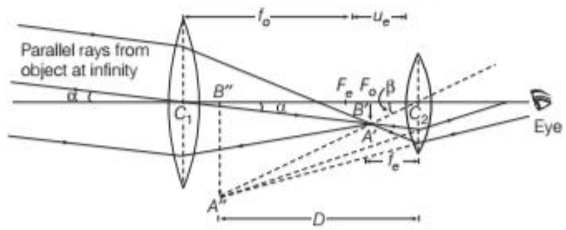
Now, $\tan\alpha = \frac{I}{f_o}$ and $\tan\beta = \frac{I}{-f_e}$

where, I is the image formed by the objective, f_o and f_e are the focal lengths of objective and eyepiece, respectively.

Substituting the values of $\tan\alpha$ and $\tan\beta$ in Eq. (i), we get

$$m = \frac{-\frac{I}{f_e}}{\frac{I}{f_o}} \quad \text{or} \quad \boxed{m = -\frac{f_o}{f_e}}$$

***Case* II When final image is formed at near point**



Angular magnification, $m = \frac{\beta}{\alpha}$

$$\Rightarrow \quad m = \frac{\tan\beta}{\tan\alpha} \quad [\because \beta \text{ and } \alpha \text{ are small}]$$

$$\Rightarrow \quad m = \frac{\frac{A'B'}{C_2B'}}{\frac{A'B'}{C_1B'}} = \frac{C_1B'}{C_2B'}$$

$$\Rightarrow \quad m = \frac{f_o}{-u_e} \quad \ldots\text{(i)}$$

Using lens formula $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$ for the eyepiece, we have

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Putting the value of $\frac{1}{u_e}$ in Eq. (i), we have

$$\boxed{m = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

EXAMPLE |5| A telescope consists of two lenses of focal lengths 20 cm and 5 cm. Obtain its magnifying power when the final image is (i) at infinity (ii) at 25 cm from the eye.

Sol. (i) When the final image is at infinity,

$$m = -\frac{f_o}{f_e} = \frac{-20}{5}$$

$\Rightarrow \quad m = -4$

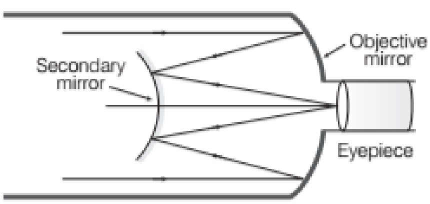
(ii) When the final image is at 25 cm from the eye, i.e. $D = 25$ cm,

$$m = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right) = \frac{-20}{5}\left(1 + \frac{5}{25}\right)$$

$\Rightarrow \quad m = -4.8$

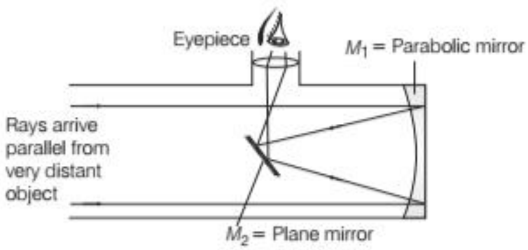
Reflecting Telescope

Reflecting telescope is also known as **Cassegrain Telescope,** which was designed by Guillaume Cassegrain, shown in figure below. Reflecting telescope is an improvement over refracting or astronomical telescope. To obtain a bright image of a distant star by refracting telescope, it is essential to have an objective of large aperture, so that it may collect more light coming from the object. But to deal with such a big lens is problem in terms of using and making and it is too costly. The same bright image of a distant object can be obtained by using a concave mirror of large aperture in place of objective.



Reflecting telescope consists of concave mirror of large aperture and large focal length (objective). A convex mirror is placed between the concave mirror and its focus. A small convex lens works as eyepiece.In the reflecting telescope, parallel rays from a distant object are intercepted and focused by a reflecting concave mirror rather than a refracting lens. One popular configuration of mirror and eyepiece is called the Newtonian reflecting type telescope, named after its designer Newton.

The parallel beam of light coming from the distant object (star) is reflected by concave parabolic mirror M_1, on the plane mirror M_2. The plane mirror M_2 is inclined at an angle of 45° to axis of the mirror M_1.



The plane mirror reflects the beam and a real image is formed in front of eyepiece. The eyepiece acts as a magnifier and the final magnified image of the distant object can be observed by the eye.

Advantages of Reflecting Telescope over Refracting Telescope

For astronomical telescope, the mirror affords several advantages over the objective lens. A mirror is easier to produce with a larger diameter, so that it can intercept rays crossing a larger area and direct them to the eyepiece.

The mirror can be made parabolic to reduce spherical aberration. Aberration is further reduced because passage through one layer of glass (the objective lens) is eliminated.

TOPIC PRACTICE 4

OBJECTIVE Type Questions

1. A prism has refractive angle 60°. When a light ray is incident on 50°, then minimum deviation is obtained. What is the value of minimum deviation?

(a) 40° (b) 45°
(c) 50° (d) 60°

2. A ray of light passes through an equilateral prism such that, the angle of incidence is equal to the angle of emergence and the latter is equal to 3/4 the angle of prism. The angle of deviation is

(a) 25° (b) 30°
(c) 45° (d) 35°

3. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is **NCERT Exemplar**

(a) 7.5° (b) 5° (c) 15° (d) 2.5°

4. The image formed by an objective of a

(a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged

5. In order to increase the angular magnification of a simple microscope, one should increase

(a) the object size
(b) the aperture of the lens
(c) the focal length of the lens
(d) the power of the lens

6. F_1 and F_2 are focal lengths of objective and eyepiece respectively, of the telescope. The angular magnification of the given telescope is equal to

(a) $\frac{F_1}{F_2}$ (b) $\frac{F_2}{F_1}$

(c) $\frac{F_1 F_2}{F_1 + F_2}$ (d) $\frac{F_1 + F_2}{F_1 F_2}$

7. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of the eyepiece are

(a) $f_o = 45$ cm and $f_e = -9$ cm
(b) $f_o = -7.2$ cm and $f_e = 5$ cm
(c) $f_o = 50$ cm and $f_e = 10$ cm
(d) $f_o = 30$ cm and $f_e = 6$ cm

8. Limitation of reflecting telescope is

(a) objective mirror focusses light inside the telescope tube
(b) objective mirror focusses light outside the telescope tube
(c) objective mirror has large focal length
(d) tube length is large

VERY SHORT ANSWER Type Questions

9. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light? Give reason. **All India 2017**

10. Write the relationship between angle of incidence i, angle of prism A and angle of minimum deviation δ_m for a triangular prism. **Delhi 2013**

11. Why should the objective lens of a compound microscope have a small focal length?

12. How will you distinguish between a compound microscope and a telescope simply by seeing it?

SHORT ANSWER Type Questions

13. What should be the position of the object relative to the biconvex lens, so that this lens behaves like a magnifying glass?

14. How does the magnification of a magnifying glass differ from its magnifying power?

15. Is it possible to increase the range of a telescope by increasing the diameter of the objective lens?

16. Draw a schematic arrangement of a reflecting telescope (Cassegrain) showing how rays coming from a distant object are received at the eyepiece. Write its two important advantages over a refracting telescope. **Delhi 2013**

17. Explain two advantages of a reflecting telescope over a refracting telescope. **CBSE 2018**

LONG ANSWER Type I Questions

18. Choose the statement as wrong or right and justify.

(i) The intensity of scattered light varies inversely as square of wavelength.
(ii) Magnification of simple microscope when final image is at infinity is given by $m = 1 - \frac{d}{f}$.
(iii) In reflecting type telescope, objective lens is replaced by convex parabolic mirror.

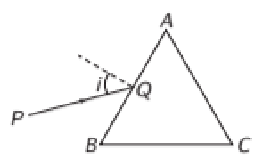
19. (i) Draw a neat labelled ray diagram of a compound microscope. Explain briefly its working.
(ii) Why must both the objective and the eyepiece of a compound microscope have short focal lengths? **All India 2010**

20. Draw a ray diagram showing the image formation by a compound microscope. Hence, obtain the expression for total magnification, when the image is formed at infinity. **Delhi 2010**

21. Draw a labelled ray diagram on a refracting telescope. Define its magnifying power and write the expression for it. Write two important limitations of a refracting telescope over a reflecting type telescope. **All India 2013**

LONG ANSWER Type II Questions

22. (i) A ray PQ of light is incident on the face AB of a glass prism ABC (as shown in the figure) and emerges out of the face AC. Trace the path of the ray. Show that $\angle i + \angle e = \angle A + \angle \delta$



where, δ and e denote the angle of deviation and angle of emergence, respectively.

Plot a graph showing the variation of the angle of deviation as a function of angle of incidence. State the condition under which $\angle\delta$ is minimum.

(ii) Find out the relation between the refractive index (μ) of the glass prism and $\angle A$ for the case, when the angle of prism (A) is equal to the angle of minimum deviation (δ_m). Hence, obtain the value of the refractive index for angle of prism $A = 60°$. **Delhi 2015**

23. Draw a ray diagram to show refraction of a ray of monochromatic light passing through a glass prism. Deduce the expression for the refractive index of glass in terms of angle of prism and angle of minimum deviation. **Delhi 2012**

24. Define magnifying power of a telescope. Write its expression.

A small telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If this telescope is used to view a 100 m high tower 3 km away, find the height of the final image, when it is formed 25 cm away from the eyepiece. **Delhi 2012**

NUMERICAL PROBLEMS

25. White light is incident on one of the refracting surface of a prism of angle 5°. If the refractive indices for red and blue colours are 1.641 and 1.659 respectively, then what will be the angular separation between these two colours when they emerge out?

26. Consider a telescope whose objective lens has a focal length of 100 cm and the eyepiece has focal length 1 cm. What will be the magnification of the given telescope?

27. Two lenses of focal lengths 6 cm and 50 cm are to be used for making a telescope. Which will you see for the objective?

28. A ray of light, incident on an equilateral glass prism ($\mu_g = \sqrt{3}$) moves parallel to the base line of the prism inside it. Find the angle of incidence for this ray. **Delhi 2012**

29. A ray PQ incident on the refracting face BA is refracted in the prism BAC and emerges from the other refracting face AC as RS, such that $AQ = AR$. If the angle of prism $\angle A = 160°$ and refractive index of the material of prism is $\sqrt{3}$, then what will be the angle of deviation of the ray?

30. The following table gives the values of the angle of deviation, for different values of the angle of incidence, for a triangular prism.

Angle of incidence	33°	38°	42°	52°	60°	71°
Angle of deviation	60°	50°	46°	40°	43°	50°

(i) For what value of the angle of incidence, is the angle of emergence likely to be equal to the angle of incidence itself ?

(ii) Draw a ray diagram, showing the passage of a ray of light through this prism, when the angle of incidence has the above value.

31. The near vision of an average person is 25 cm. To view an object with an angular magnification of 10, what should be the power of the microscope? **NCERT Exemplar**

32. You are given two converging lenses of focal length 1.25 cm and 5 cm to design a compound microscope. If it is desired to have a magnification of 30, then find out the separation between the objective and eyepiece. **Delhi 2015**

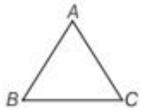
33. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece? **NCERT**

34. A telescope consists of two thin lenses of focal lengths 0.3 m and 3 cm, respectively. It is focused on moon which subtends an angle of 0.5° at the objective. Then, what will be the angle subtended at the eye by the final image?

35. A small telescope has an objective lens of focal length 150 cm and eyepiece of focal length 5 cm. What is the magnifying power of the telescope for viewing distant objects in normal adjustments?

If this telescope is used to view a 100 m tall tower 3 km away, then what is the height of the tower formed by the objective lens? **Delhi 2015**

36. (i) A ray of light incident of face AB of an equilateral glass prism, shows minimum deviation of 30°. Calculate the speed of light through the prism.



(ii) Find the angle of incidence at face AB, so that the emergent ray grazes along the face AC. **Delhi 2017**

37. For a glass prism($\mu = \sqrt{3}$), the angle of minimum deviation is equal to the angle of the prism. Find the angle of the prism. **NCERT Exemplar**

38. (i) Draw a labelled ray diagram showing the formation of a final image by a compound microscope at least distance of distinct vision.

(ii) The total magnification produced by a compound microscope is 20. The magnification produced by the eyepiece is 5. The microscope is focused on a certain object. The distance between the object and eyepiece is observed to be 14 cm. If least distance of distinct vision is 20 cm, calculate the focal length of the object and the eyepiece. **Delhi 2014**

39. A compound microscope uses an objective lens of focal length 4 cm and eyepiece lens of focal length 10 cm . An object is placed at 6 cm from the objective lens. Calculate the magnifying power of the compound microscope. Also, calculate the length of the microscope. **All India 2011**

40. (i) A giant reflecting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece lens of focal length 1.0 cm is used, find the angular magnification of the telescope.

(ii) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^{6} m and the radius of the lunar orbit is 3.8×10^{8} m. **All India 2015, All India 2011; NCERT**

HINTS AND SOLUTIONS

1. (a) Given, incidence angle, $i = 50^\circ$

Refraction angle, $A = 60^\circ$

Minimum deviation, $\delta = 2i - A = 50^\circ \times 2 - 60^\circ = 40^\circ$

2. (b) Given, equilateral prism i.e., $A = 60^\circ$

$$i = e = \frac{3}{4}A = \frac{3}{4}\times 60^\circ = 45^\circ$$

From relation, $A + D = i + e$

We have, $60^\circ + D = 2\times 45^\circ$

$\Rightarrow \quad D = 90^\circ - 60^\circ = 30^\circ$

3. (a) Since, deviation $\delta = (\mu - 1)\,A = (1.5 - 1)\times 5^\circ = 2.5^\circ$

By geometry, angle of refraction by first surface is 5°.

But $\delta = \theta - r$, so, we have, $2.5^\circ = \theta - 5^\circ$ on solving $\theta = 7.5^\circ$.

4. (c) Objective of a compound microscope is a convex lens. Convex lens forms real and enlarged image when an object is placed between focus and radius of curvature.

5. (d) For least distance of distinct vision, the angular magnification of simple microscope is

$$M = 1 + \frac{D}{f} \Rightarrow M = 1 + DP \qquad \left(\because \text{Power}(P) = \frac{1}{f}\right)$$

and for normal adjustment $M = \frac{D}{f} \Rightarrow M = DP \Rightarrow M \propto P$.

6. (a) Given, $f_o = F_1,\ f_e = F_2$

We know, angular magnification for telescope

$$|M| = \left|\frac{f_o}{f_e}\right| = \left|\frac{F_1}{F_2}\right| \Rightarrow \frac{F_1}{F_2}$$

7. (d) For telescope $|m| = \frac{|f_o|}{|f_e|} = 5$ ...(i)

and length of the telescope

$L = |f_o| + |f_e| = 36$ ...(ii)

From Eqs. (i) and (ii),

$\Rightarrow f_e = 6\text{ cm}$ and $f_o = 30\text{ cm}$

8. (a) The main limitation of reflecting telescope is that the objective mirror focusses light inside the telescope tube.

9. Wavelength of violet light is smaller than that of red light. Also, angle of minimum deviation,

$$\delta_m = (\mu - 1)A$$

$\Rightarrow \quad \delta_m \propto \mu$

As, $\quad \mu_R < \mu_V$

$\Rightarrow \quad (\delta_m)_R < (\delta_m)_V$

As deviation is less for red light, hence angle of deviation decreases.

10. The relation between the angle of incidence i, angle of prism A and the angle of minimum deviation δ_m, for a triangular prism is given as $i = \frac{A + \delta_m}{2}$.

11. The angular magnification of eyepiece is $\left(1 + \frac{d}{f_e}\right)$.

Hence, as f_e decreases angular magnification increases.

Also, the magnification of the object lens is $\frac{v}{u}$.

The object lies close the focus of the objective lens $u = f_o$. Therefore, to increase the magnification f_o should be small.

12. In compound microscope objective lens has smaller aperture and smaller focal length than the eyepiece, while in telescope, the objective has a larger aperture and larger focal length than the eyepiece.

13. Whenever object is placed within the focus of the biconvex lens, we will obtain enlarged image, hence the biconvex lens behaves like a magnifying lens.

14. The **magnification** of a magnifying glass depends upon, where it is placed between the user's eye and the object being viewed and the total distance between them, while the **magnifying power** is equivalent to angular magnification.

15. By increasing the diameter of the objective lens, we can increase the range of the telescope because as the diameter of lens increases, the area covered by the lens also increases, i.e. lens is able to focus on a large area there by helping us to view the object better.

16. Refer to text on page 387.

17. Advantages of reflecting telescope over refracting telescope

(i) In reflecting telescope, image formed is free from chromatic aberration defect. So, it is sharper than image formed by a retracting type telescope.

(ii) A mirror is easier to produce with a large diameter, so that it can intercept rays crossing a large area and direct them to the eye-piece.

18. (i) Wrong; as it varies $I \propto \frac{1}{\lambda^4}$.

(ii) Refer to text on pages 383 and 384.

(iii) Refer to text on pages 387.

19. (i) Refer to text on pages 384 and 385.

(ii) f_o and f_e of compound microscope must be small, so as to have large magnifying power as

$$m = \frac{-1}{f_o}\left(1 + \frac{D}{f_e}\right)$$

20. Refer to text on pages 384 and 385.

21. Refer to text on page 386 and 387.

Magnifying power (in normal adjustment) of a reflecting telescope is the ratio of the focal length of concave reflector and the focal length of eyepiece.

$$m = \frac{f}{f_e} = \frac{R/2}{f_e}$$

Limitations of refracting telescope over a reflecting type telescope

(i) Refracting telescope suffers from chromatic aberration as it uses large sized lenses.

(ii) It is also difficult and expensive to make such large sized lenses.

22. (i) Refer to text on pages 382 and 383.

(ii) Refer to text on pages 382 and 383.

Since, $\angle A = 60^\circ$ [given]

$$\therefore \quad \mu = \frac{\sin\left(\frac{60^\circ + 60^\circ}{2}\right)}{\sin(60^\circ/2)}$$

$$= \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}}{2} \times \frac{2}{1}$$

$$= \sqrt{3} = 1.732$$

23. Refer to text on pages 382 and 383.

24. The magnifying power of a telescope is equal to the ratio of the visual angle subtended at the eye by final image formed at least distance of distinct vision to the visual angle subtended at naked eye by the object at infinity.

Magnification, $m = \frac{I}{O} = \frac{v_0}{u_0} = \frac{f_0}{u_0}$

$$\Rightarrow \quad \frac{I}{100} = \frac{150 \times 10^{-2}}{3 \times 10^3}$$

$$\Rightarrow \quad I = 5 \times 10^{-2} \text{ m} = 5 \text{ cm}$$

25. Angle of prism, $A = 5^\circ, \mu_R = 1.641, \mu_B = 1.659$

As we know that, $\delta = (\mu - 1)A$

So, $\delta_B = (\mu_B - 1)\, A$ and $\delta_R = (\mu_R - 1)\, A$

$\therefore \quad \delta_B - \delta_R = (\mu_B - \mu_R)A = (1.659 - 1.641) \times 5$

$= 0.09^\circ$

26. Magnification of telescope is given by

$$m = \frac{f_o}{f_e} = \frac{100}{1} = 100$$

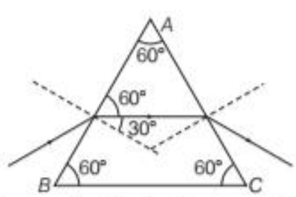
27. Yes, these lenses can be used for making a telescope. Since, the objective lens has large aperture and focal length, hence the lens having focal length will be used as objective lens.

28. To draw the ray diagram for the refraction from the prism. Following things should be kept in mind.

(i) Draw normal to the point of incidence.

(ii) Consider each boundary of the prism as separate interface and draw the ray diagram for the refraction taking place.

The reflection of light through prism is shown below.



By geometry, angle of refraction, $r = 30^\circ$

Refractive index, $\mu = \sqrt{3}$ $\left[\text{from Snell's law, } \mu = \frac{\sin i}{\sin r}\right]$

$$\Rightarrow \quad \sin i = \mu \sin r = (\sqrt{3})\sin(30^\circ) = \frac{\sqrt{3}}{2}$$

Angle of incidence, $i = 60^\circ = \frac{\pi}{3} \Rightarrow i = \frac{\pi}{3}$

29. Refer to the Example 2 on pages 383.

30. (i) $i = 52^\circ$, when prism is adjusted at an angle of minimum deviation, then angle of incidence is equal to the angle of emergence.

Hence, $r = 0$

This ray pass unrefracted at AC interface and reaches AB interface. Here, we can see angle of incidence becomes 30°.

Thus, applying Snell's law, $\frac{\sin 30^\circ}{\sin e} = \frac{\mu_a}{\mu_g} = \frac{1}{\sqrt{3}}$

$$\sin e = \sqrt{3} \times \sin 30^\circ = \frac{\sqrt{3}}{2}$$

Thus, $e = 60^\circ$

31. The least distance of distinct vision of an average person, (i.e. D) is 25cm, in order to view an object with magnification of 10.

Here, $v = D = 25$ cm and $u = f$

But the magnification, $m = v/u = D/f$

$$\therefore \quad m = \frac{D}{f}$$

$$\Rightarrow \quad f = \frac{D}{m} = \frac{25}{10} = 2.5 = 0.025 \text{ m}$$

and $P = \frac{1}{0.025} = 40$ D $\quad \left[\because P = \frac{1}{f}\right]$

This is the required power of lens .

32. Given, $f_o = 1.25$ cm, $f_e = -5$ cm

Magnification, $m = 30$, $D = 25$ cm

If the object is very close to the principal focus of the objective and the image formed by the objective is very close to eyepiece, then magnifying power of a microscope is given by

$$m = -\frac{L}{f_o} \cdot \frac{D}{f_e}$$

$$\Rightarrow \quad 30 = \frac{L}{1.25} \cdot \frac{25}{5}$$

$$\Rightarrow \quad L = \frac{125 \times 30 \times 5}{25 \times 100}$$

$$\Rightarrow \quad L = \frac{25 \times 30}{100} \Rightarrow L = \frac{30}{4}$$

$$\Rightarrow \quad L = 7.5 \text{ cm}$$

This is a required separation between the objective and the eyepiece.

33. Given, focal length of objective lens, $f_o = 144$ cm

Focal length of eyepiece, $f_e = 6$ cm

Magnifying power of the telescope in normal adjustment (i.e. when the final image is formed at ∞),

$$m = -\frac{f_o}{f_e} = -\frac{144}{6} = -24$$

$\therefore$ Separation between lenses,

$$L = f_o + f_e = 144 + 6 = 150 \text{ cm}$$

34. Since, $m = \frac{\tan\beta}{\tan\alpha} = \frac{\beta}{\alpha} = \frac{f_o}{f_e}$

$$\therefore \quad \frac{\beta}{0.5^\circ} = \frac{0.3}{0.03} = 5^\circ$$

35. When final image is at D, then

magnifying power, $m = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

In normal adjustment, $m = \frac{-f_o}{f_e}$

For telescope,

focal length of objective lens, $f_o = 150$ cm

Focal length of eye lens, $f_e = 5$ cm

When final image forms at D, i.e. 25 cm, then

magnification, $m = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

$$= \frac{-150}{5}\left(1 + \frac{5}{25}\right) = \frac{-150}{5} \times \frac{6}{5}$$

$$\Rightarrow \quad m = -36$$

Let height of final image be h cm.

$$\Rightarrow \quad \tan\beta = \frac{h}{25} \text{ and } \tan\alpha = \frac{100 \text{ m}}{3000 \text{ m}} = \frac{1}{30}$$

where, β = visual angle formed by final image at eye and α = visual angle subtended by object at objective.

But $m = \frac{\tan\beta}{\tan\alpha} \Rightarrow -36 = \frac{\left(\frac{h}{25}\right)}{\left(\frac{1}{30}\right)}$

$$\Rightarrow \quad -36 = \frac{h}{25} \times 30 \Rightarrow -36 = \frac{6h}{5}$$

$$\Rightarrow \quad h = \frac{-36 \times 5}{6}$$

$$h = -30 \text{ cm}$$

Negative sign indicates inverted image.

36. (i) Given, angle of minimum deviation, $\delta_m = 30^\circ$

$\therefore$ Angle of prism, $A = 60^\circ$

By prism formula, reflected index,

$$\mu = \frac{\sin \frac{\delta_m + A}{2}}{\sin A/2} = \frac{\sin \frac{30^\circ + 60^\circ}{2}}{\sin 30^\circ} = \frac{\sin 45^\circ}{\sin 30^\circ}$$

$$= \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

Also, $\mu = \frac{\text{speed of light in vacuum } (c)}{\text{speed of light in prism } (v)}$

$$\Rightarrow \quad v = c/\mu = (3 \times 10^8/\sqrt{2}) \text{ m/s}$$

Hence, speed of light through prism is $(3 \times 10^8/\sqrt{2})$ m/s

(ii) Critical angle i_c is given as

$$\sin i_c = \frac{1}{\sqrt{2}} \quad \left[\because \sin i_c = \frac{1}{\mu}\right]$$

$$\Rightarrow \quad i_c = 45^\circ$$
$$A = r + i_c = 60^\circ$$
$$\Rightarrow \quad r = 60^\circ - 45^\circ = 15^\circ$$

Using Snell's law, $\dfrac{\sin i}{\sin r} = \sqrt{2}$

$$\Rightarrow \quad \sin i = \sqrt{2}\sin r = \sqrt{2} \times \sin 15^\circ$$
$$\therefore \quad i = \sin^{-1}(\sqrt{2}\sin 15^\circ)$$

37. The relationship between refractive index, prism angle A and angle of minimum deviation δ_m is given by

$$\mu = \frac{\sin[(A+\delta_m)/2]}{\sin(A/2)}$$

Given, $\delta_m = A$

Substituting the value of δ_m, we have

$$\therefore \quad \mu = \frac{\sin A}{\sin(A/2)}$$

On solving, we have, $\mu = \dfrac{2\sin\dfrac{A}{2}\cos\dfrac{A}{2}}{\sin\dfrac{A}{2}} = 2\cos\dfrac{A}{2}$

For the given value of refractive index, $\mu = \sqrt{3}$, we have

$$\cos\frac{A}{2} = \frac{\sqrt{3}}{2} \text{ or } \frac{A}{2} = 30^\circ \text{ or } A = 60^\circ$$

This is the required value of prism angle.

38. (i) Refer to text on page 384.

(ii) Given, magnification, $m = 20$

Magnification of eyepiece, $m_e = 5$

Least distance vision, $D = 20$ cm

Distance between the object and eyepiece,

$$L = 14 \text{ cm}$$

We know that, magnification, $m = m_e \times m_o$

$$\Rightarrow \quad m_o = \frac{m}{m_e} = \frac{20}{5} = 4$$

As, $\quad m_e = 1 + \dfrac{D}{f_e}$

where, f_e is focal length of eyepiece.

$$\Rightarrow \quad 5 = 1 + \frac{20}{f_e} \Rightarrow f_e = 5 \text{ cm}$$

Using lens formula for eyepiece,

$$\frac{1}{u_e} = \frac{-1}{20} - \frac{1}{5} = \frac{-5}{20} = \frac{-1}{4}$$

$$\Rightarrow \quad u_e = -4 \text{ cm (object distance for eyepiece)}$$
$$\Rightarrow \quad L = v_o + |u_e|$$
$$\Rightarrow \quad v_o = L - |u_e|$$
$$= 14 - 4 = 10 \text{ cm}$$

Magnification produced by object, $m_o = -\dfrac{v_o}{u_o}$

Object distance for object,

$$u_o = \frac{-v_o}{m_o} = \frac{-10}{4} = -2.5 \text{ cm}$$

Using lens formula for object,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{10} - \frac{1}{-2.5} = \frac{1}{10} + \frac{1}{2.5}$$
$$f_o = 2 \text{ cm}$$

39. For compound microscope, $f_o = 4$ cm, $f_e = 10$ cm,

$$u_o = -6 \text{ cm}, v_e = -D = -25 \text{ cm}$$

For objective lens, $\dfrac{1}{f_o} = \dfrac{1}{v_o} - \dfrac{1}{u_o} \Rightarrow \dfrac{1}{4} = \dfrac{1}{v_o} + \dfrac{1}{6}$

$$\Rightarrow \quad \frac{1}{v_o} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12} \Rightarrow v_o = 12 \text{ cm}$$

$\therefore$ Magnifying power, $m = -\left(\dfrac{v_o}{u_o}\right)\left(1 + \dfrac{D}{f_e}\right)$

$$= -\left(\frac{12}{6}\right)\left(1 + \frac{25}{10}\right) = -2\left(\frac{7}{2}\right) = -7$$

Length of microscope $= |v_o| + |u_e|$

where, $\quad v_o = 12$ cm

For eye lens, $v_e = -25$ cm, $f_e = 10$ cm, $u_e = ?$

$$\therefore \quad \frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$
$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{10}$$
$$\Rightarrow \quad \frac{1}{u_e} = \frac{-2-5}{50} = -\frac{7}{50}$$
$$\Rightarrow \quad u_e = -7.14 \text{ cm}$$

$\therefore$ Length of microscope $= |v_o| + |u_e|$

$$= 12 + 7.14 = 19.14 \text{ cm}$$

40. (i) For astronomical telescope,

$$f_o = 15 \text{ m} = 1500 \text{ cm}, f_e = 1 \text{ cm}$$

Angular magnification, $m = -\dfrac{f_o}{f_e} = -\dfrac{15 \times 100 \text{ cm}}{1 \text{ cm}}$

$$= -1500$$

(ii) Given, diameter, $D = 3.48 \times 10^6$ m,

$$f_o = 3.8 \times 10^8 \text{ m}$$

Let α be the angle subtended by the moon at objective.

$$\therefore \quad \alpha = \frac{D}{\text{Radius of lunar orbit}}$$
$$\alpha = \frac{3.48 \times 10^6 \text{ m}}{3.8 \times 10^8 \text{ m}} \quad \text{...(i)}$$

Also, then angle subtended by image formed by objective on itself,

$$\alpha = \frac{d}{f_o} \quad \text{...(ii)}$$

where, $\quad d$ = diameter of image.

From Eqs. (i) and (ii), we get

$$\frac{3.48 \times 10^6}{3.8 \times 10^8} = \frac{d}{1500} \quad \text{[given]}$$

$$\therefore \quad d = \frac{1500 \times 3.48 \times 10^6}{3.8 \times 10^8} = 13.7 \text{ cm}$$

SUMMARY

- **Reflection of Light** It is the phenomenon of change in the path of light without any change in medium.
- **Laws of Reflection** Incident ray, reflected ray and the normal to the reflecting surface at the point of incidence, all lies in the same plane. Angle of incidence is always equal to the angle of reflection.
- **Mirror formula** is given by, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.
- **Linear Magnification** The ratio of size of the image formed by the spherical mirror to the size of the object is called linear magnification,

  i.e. $m=\frac{I}{O}=\frac{-v}{u}$
- **Refraction** It is the phenomenon of change in the path of light as it goes from one medium to another medium.
- **Laws of Refraction** Incident ray, refracted ray and the normal to the refracting surface at the point of incidence, all lies in the same plane.

  According to second law, $(\sin i / \sin r) = {}_1\mu_2$. This is called Snell's law of refraction.
- **Refractive Index** It is equal to the ratio of speed of light in vacuum to the speed of light in the material.
- **Principle of Reversibility of Light** When a light rays, after suffering any number of reflections and refractions, its final path is reversed and it travels back along its entire initial path.
- **Expression for Lateral Displacement** $D=\frac{t\sin(i_1-r_1)}{\cos r_1}$
- **Apparent Depth and Real Depth** ${}^a\mu_w=\frac{\text{Real depth}}{\text{Apparent depth}}$
- **Critical Angle** The angle of incidence in denser medium corresponding to which angle of refraction in rarer medium is 90°.
- **Total Internal Reflection** (TIR) The ray on the interface of two media should travel in the denser medium. The angle of incidence should be greater than the critical angle for the two media.
- **Refraction at Spherical Surfaces** The equation which holds the good for any curved spherical surface is given by $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$
- **Lens** It is a transparent medium bounded by two surfaces of which one or both surfaces are spherical. It is of two types.

  Convex lens is thicker at the centre and thinner at its end.

  Concave lens is thinner at the centre and thicker at its end.
- **Lens formula** $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$
- **Lens Maker's formula** $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$
- **Power of a Lens** It is the ability to converge or diverge the rays of incident light.

  $$P=\frac{1}{f(\text{in m})}$$

  Also, $$P=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
- **Prism** A prism is a portion of transparent medium bounded by two plane faces inclined to each other at a suitable angle.
- **Refraction of Light through a Prism** The relation between angle of deviation and angle of prism is $\delta=(\mu-1)A$.
- **Prism Formula** It is given by, $\mu=\frac{\sin\left[\frac{A+\delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}$
- **Angle of minimum deviation** $\delta_m=(\mu-1)A$
- **Simple Microscope** It forms the large image of close and minute objects. It is a converging lens of small focal length.
- **Compound Microscope** It consists of two convex lenses coaxially separated by some distance. One is objective and another is eyepiece.
- **Astronomical Telescope** It has two convex lenses coaxially separated by some distance, which is used for observing distinct images of heavenly by bodies like stars, planet, etc.
- **Refracting and Reflecting Telescope** Refracting telescope is used for observing the distinct images of heavenly bodies like stars, planets, etc.

  Reflecting telescope is an improvement over refracting telescope.

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Relation between focal length (f) and radius of curvature (R) of a spherical mirror is
   (a) $R = f/2$ (b) $f = 3R$
   (c) $f = R/2$ (d) $f = R/4$

2. A convex mirror has focal length 20 cm. If an object is placed 20 cm away from the pole of mirror, then what is the distance between image formed and pole?
   (a) 40 cm (b) 10 cm
   (c) 20 cm (d) At infinity

3. In total internal reflection,
   (a) light ray travelling through a denser medium is completely reflected back to denser medium
   (b) light ray travelling through a denser medium is completely refracted to rare medium
   (c) light ray is partially reflected back to denser medium and partially refracted to rare medium
   (d) light ray is absorbed completely by denser medium

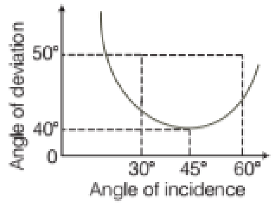
4. Ray of light transmitted from glass ($n = 3/2$) to water ($n = 4/3$). What is the value of critical angle?
   (a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\sqrt{\frac{8}{9}}$
   (c) $\sin^{-1}\left(\frac{8}{9}\right)$ (d) $\sin^{-1}\left(\frac{5}{7}\right)$

5. Two convex and concave lens are in contact and having focal length 12 cm and 18 cm, respectively. Focal length of joint lens will be
   (a) 50 cm (b) 45 cm (c) 36 cm (d) 18 cm

6. Two lenses are kept in contact with powers + 2 D and – 4 D. The focal length of this combination will be
   (a) + 50 cm (b) – 50 cm
   (c) – 25 cm (d) + 25 cm

7. A thin lens of glass ($\mu = 1.5$) of focal length ± 10 cm is immersed in water ($\mu = 1.33$). The new focal length is
   (a) 20 cm (b) 40 cm (c) 48 cm (d) 12 cm

8. A plot of angle of deviation D *versus* angle of incidence for a triangular prism is shown below. The angle of incidence for which the light ray travels parallel to the base is



   (a) 30° (b) 60°
   (c) 45° (d) Data insufficient

9. An equilateral prism is in condition of minimum deviation. If incidence angle is 4/5 times of prism angle, then minimum deviation angle is
   (a) 72° (b) 60° (c) 48° (d) 36°

10. Advantage of reflecting telescopes are
   (a) no chromatic aberration
   (b) parabolic reflecting surfaces are used
   (c) weighs of mirror are much less than a lens of equivalent optical quality
   (d) All of the above

11. When a wave undergoes reflection at an interface from rarer to denser medium, then change in its phase is **CBSE 2020**
   (a) $\frac{\pi}{2}$ (b) zero (c) π (d) $\frac{\pi}{4}$

12. A bi-convex lens of focal length f is cut into two identical plano-convex lenses. The focal length of each part will be **CBSE 2020**
   (a) f (b) $\frac{f}{2}$ (c) $2f$ (d) $4f$

13. biconcave lens of power P vertically splits into two identical plano-concave parts. The power of each part will be **CBSE 2020**
   (a) $2P$ (b) $\frac{P}{2}$ (c) P (d) $\frac{P}{\sqrt{2}}$

ASSERTION AND REASON

Directions (Q. Nos. 14-19) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

14. **Assertion** A ray of light incident along the normal to the plane mirror retraces its path after reflection from the mirror.

Reason A ray of light along the normal has angle of incidence as $\pi/2$ and hence, it retraces its own path after reflection from mirror.

15. **Assertion** Refractive index of glass with respect to air is different for red light and violet light.

Reason Refractive index of a pair of media depends on the wavelength of light used.

16. **Assertion** Propagation of light through an optical fibre is due to total internal reflection taking place at the core-clade interface.

Reason Refractive index of the material of the core of the optical fibre is greater than that of air.

17. **Assertion** The refractive index of diamond is $\sqrt{6}$ and that of liquid is $\sqrt{3}$. If the light travels from diamond to the liquid, it will initially reflected when the angle of incidence is 45°.

Reason $\mu = \frac{1}{\sin C}$, where μ is the refractive index of diamond with respect to liquid.

18. **Assertion** Convergent lens property of converging remains same in mediums.

Reason Property of lens whether the ray is diverging or converging depends on the surrounding medium.

19. **Assertion** By roughening the surface of a glass at sheet its transparency can be reduced.

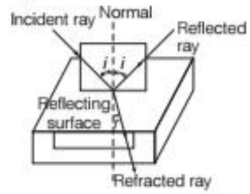
Reason Glass sheet with rough surface absorbs more light.

CASE BASED QUESTIONS

Directions (Q.No. 20) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

20. **Refraction of Light**

Refraction involves change in the path of light due to change in the medium.



When a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other. The direction of propagation of an obliquely incident ray of light, that enters the other medium, changes at the interface of two media. This phenomenon is called refraction of light.

(i) Which of the following quantity remains unchanged after refraction?

(a) Speed of light
(b) Intensity of light
(c) Wavelength of light
(d) Frequency of light

(ii) A ray of light strikes an air-glass interface at an angle of incidence $(i = 60°)$ and gets refracted at an angle of refraction r. On increasing the angle of incidence $(i > 60°)$, the angle of refraction r

(a) decreases
(b) remains same
(c) is equal to 60°
(d) increases

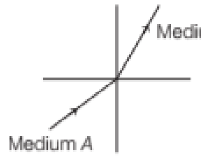
(iii) When an object lying in a denser medium is observed from rarer medium, then real depth of object is

(a) more than that observed
(b) less than that observed
(c) equals to observed depth
(d) depends on angle of vision

(iv) For the same angle of incidence, the angles of refraction in media P, Q and R are 35°, 25° and 15°, respectively. Which of the following relation hold true for the velocity of light in medium P, Q and R?

(a) $v_P < v_Q < v_R$
(b) $v_P < v_R < v_Q$
(c) $v_P > v_Q > v_R$
(d) $v_P > v_R > v_Q$

(v) A light ray enters from medium A to medium B as shwon in figure. The refractive index of medium B relative to A will be



(a) greater than unity (b) less than unity
(c) equal to unity (d) zero

VERY SHORT ANSWER Type Questions

21. At what angle, is a ray of light falling normally on a mirror reflected?

22. Does size of mirror affect the nature of the image?

23. Why are danger signals red in colour ?

24. Why does a convex lens of glass $\mu = 1.5$ behave as a diverging lens when immersed in carbon disulphide of $\mu = 1.65$?

25. Why does rising sun appear oval shaped?

SHORT ANSWER Type Questions

26. Where should an object be placed from a convex lens to form an image of the same size? Can it happen in case of a concave lens?

27. Derive the expression for the effective focal length of two thin lenses in contact.

28. Discuss refraction of monochromatic light through a prism and derive its relation.

29. A 4 cm tall light bulb is placed at a distance of 8.30 cm from a double convex lens having a focal length of 15.2 cm. Calculate the position and size of the image of the bulb.

LONG ANSWER Type I Questions

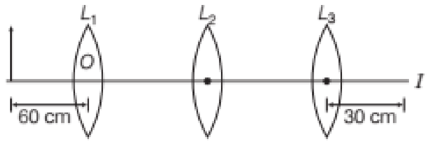
30. Minimum deviation suffered by violet, yellow and red beams passing through an equilateral transparent prism are 39.2°, 38.7° and 38.4°, respectively. Calculate the dispersive power in the medium.

31. A beam of light strikes a glass sphere of diameter 15 cm converging towards a point 30 cm behind the pole of the spherical surface. Find the position of the image, if μ of glass is 1.5.

LONG ANSWER Type II Questions

32. (i) Explain with reason, how the power of a diverging lens changes when (a) it is kept in a medium of refractive index greater than that of the lens. (b) incident red light is replaced by violet light.

(ii) Three lenses L_1, L_2 and L_3 each of focal length 30 cm are placed coaxially as shown in the figure. An object is held at 60 cm from the optic centre of lens L_1. The final real image is formed at the focus of L_3. Calculate the separation between (a) (L_1 and L_2) and (b) (L_2 and L_3). **All India 2017 C**



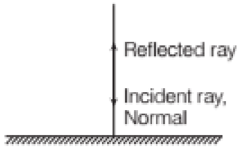
33. (i) Deduce the expression by drawing a suitable ray diagram for the refractive index of a triangular glass prism in terms of the angle of minimum deviation (D) and the angle of prism (A).
Draw a plot showing the variation of the angle of deviation with the angle of incidence.

(ii) Calculate the value of the angle of incidence when a ray of light incident on one face of an equilateral glass prism produces the emergent ray, which just grazes along the adjacent face. Refractive index of the prism is $\sqrt{2}$. **All India 2017C**

ANSWERS

1. (c)	2. (b)	3. (c)	4. (c)	5. (c)
6. (b)	7. (b)	8. (c)	9. (d)	10. (d)
11. (c)	12. (c)	13. (b)		

14. (c) Angle of incidence = Angle between incident ray and normal to the mirror = 0°



$\Rightarrow$ Angle of reflection = 0° (from laws of reflection)
Hence, the reflected ray retraces its path along the normal at an angle 0° with normal.

15. (*a*) Refractive index of any pair of media is inversely proportional to wavelength of light.

Hence, $\lambda_v < \lambda_r$

$\Rightarrow \mu_v > \mu_r$

where, λ_v and λ_r are the wavelengths of violet and red light and μ_r and μ_v are refractive index of violet and red light.

16. (*b*) Optical fibre communication is based on the phenomenon of total internal reflection at core-clad interface.

The refractive index of the material of the cladding, hence, light striking at core-cladding interface gets totally internally reflected. The light undergoes and reaches the other end of the fibre.

17. (*a*) Refractive index of diamond w.r.t. liquid

$${}^{l}\mu_d = \frac{1}{\sin C} = \frac{\mu_d}{\mu_l}$$

$$\Rightarrow \frac{\sqrt{6}}{\sqrt{3}} = \frac{1}{\sin C}$$

$$\Rightarrow \sin C = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

$$\therefore C = 45^\circ$$

18. (*d*) In air or water a convex lens made of glass behaves as a convergent lens but when it is placed in carbon disulfide, it behaves as a divergent lens. Therefore, when a convergent lens is placed inside a transparent medium having refractive index greater than that of material of lens, it behaves as a divergent lens.

19. (*c*) When glass surface is made rough, then light incident on it is scattered in different directions. Due to which its transparency decreases. There is no effect of roughness on absorption of light.

20. (i) (*d*) Refraction does not change the frequency of light.

(ii) (*d*) From Snell's law of refraction,

$${}^{a}\mu_g = \frac{\sin i}{\sin r} = \text{constant} \quad \ldots\text{(i)}$$

Since, angle of incidence increase, the angle of refraction has to increase. So, that the ratio $\left(\frac{\sin i}{\sin r}\right)$ is a constant according to Eq. (i).

(iii) (*a*) When an object lying in a denser medium is observed from rarer medium, then real depth of object is more than that observed depth.

(iv) (*c*) As, $\mu = \frac{\sin i}{\sin r}$ ...(i)

or $\mu \propto \frac{1}{\sin r}$

$\Rightarrow \mu$ is maximum for R, since r is minimum and hence, $\sin r$ is minimum.

Also, $\mu = \frac{c}{v}$ ...(ii)

Therefore, if μ is maximum, v is minimum, i.e. velocity of light is minimum in medium R and order of velocity will be $v_P > v_Q > v_R$

(v) (*a*) We see that, ray of light bends towards the normal as we go from medium A to medium B. And we know that, when ray goes from rarer to denser medium, it bends towards normal.

So, that means refractive index of B is greater than A. Thus, refractive index of B relative than A = Refractive index of B/Refractive index of A. Since, Refractive index of B > Refractive of A. Therefore, refractive index of B relative than $A > 1$.

21. A ray of light which is incident normally on a mirror, is reflected along the same path, i.e. the angle of incidence as well as the angle of reflection is zero.

22. Nature of the image is independent of the size of mirror. Image can be real or virtual depending upon the position of object.

23. The colour red is used for danger signals because red light is scattered the least by air molecules. The effect of scattering is inversely, related to the fourth power of the wavelength, i.e. $I \propto \frac{1}{\lambda^4}$ of colour, so red light is able to travel the longest distance through fog, rain and the alike.

24. Here, $\mu = \frac{\mu_g}{\mu_c} = \frac{1.5}{1.65} < 1$

$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ Hence, it becomes negative.

So, it behaves like a diverging lens.

25. Sun appears oval shaped due to atmospheric refraction.

26. Refer to the text on page 374.

27. Refer to the text on page 375.

28. Refer to the text on pages 382 and 383.

29. $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{15.2} = \frac{1}{8.30} + \frac{1}{d_i}$

$$0.0658 = 0.120 + \frac{1}{d_i}$$

$$\Rightarrow -0.0547 = \frac{1}{d_i}$$

$$-18.3 \text{ cm} = d_i$$

Also here, $d_i = -18.3$ cm

$$\therefore \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$\Rightarrow \frac{h_i}{(4.00)} = -\frac{(-18.3)}{8.30}$$

$$h_i = \frac{-(4.00)\cdot(-18.3)}{8.30}$$

$$h_i = 8.81 \text{cm}$$

30. $\therefore$ Dispersive power $= \dfrac{\mu_v - \mu_r}{\mu_y - 1} = \dfrac{\left(\dfrac{\delta_v}{A}\right) - \left(\dfrac{\delta_r}{A}\right)}{\left(\dfrac{\delta_y}{A}\right)}$

$[\because \delta = (\mu - 1)A]$

$= \dfrac{39.2 - 38.4}{38.7} = 0.0204$

31. Here, $\mu_1 = 1, \mu_2 = 1.5, u = -\infty, R = \dfrac{15}{2} = 7.5\,\text{cm}$

$\therefore$ Using $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

32. (i) (a) Refer to text on page 373.

(b) Power of a lens increases if red light is replaced by violet light because

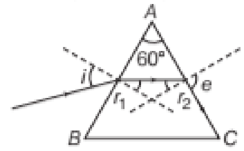
$$P = \frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As refractive index is maximum for violet light in visible region of spectrum.

(ii) Refer to Q. 34 on page 378.

33. (i) Refer to text on pages 382 and 383.

(ii) Given, the emergent ray grazes along the face AC,



$$e = 90^\circ$$

$$\mu = \sqrt{2}$$

$$\frac{\sin i}{\sin r_1} = \mu = \frac{\sin e}{\sin r_2}$$

$$\Rightarrow \quad \frac{\sin 90^\circ}{\sin r_2} = \sqrt{2}$$

i.e. $\quad \sin r_2 = \dfrac{1}{\sqrt{2}}$ or $r_2 = 45^\circ$

$$\Rightarrow \quad r_1 + r_2 = \angle A = 60^\circ$$

$$r_1 = 60 - r_2 = 15^\circ$$

$$\Rightarrow \quad \frac{\sin i}{\sin 15^\circ} = \sqrt{2}$$

$$\Rightarrow \quad i = 21.47^\circ$$